The Radiosity Method in Optical Remote Sensing of Structured 3-D Surfaces

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Abstract

The radiosity method is a mathematical concept to describe the scattering of light between ideally diffuse (Lambertian) surfaces. The method presented takes reflections, transmission and multiple scattering into account. Algorithms to find view factors and to solve the radiosity equations using the Gauss-Seidel iteration method are described. An example for a layered plant canopy model shows the relation between the radiosity method and radiative transfer. The application of the radiosity method to remote sensing problems of 3-D surfaces, e.g. calculation of a BRDF including internal shadowing effects is shown. Numerical results of radiosity calculations are compared with equivalent radiative transfer results. We conclude that the radiosity method is a valuable tool to model the transport of light in vegetative canopies as well as a tool to evaluate bidirectional reflectance characteristics of discrete leaf canopy structures, such as angular reflectance signatures.

1 Introduction

In the past three decades many optical sensors have been placed on satellites in orbit and were flown on airborne platforms to measure the visible and near infrared radiation emitted from the Earths land and water surfaces. Great efforts have been made to analyze and understand the obtained images. Models of vegetated surfaces have been used to link the spectral, temporal, spatial and angular signatures to biophysical parameters like leaf area index, leaf size, leaf angle distribution, biomass, plant stress, plant architecture, etc. Most of the existing models are based on radiative transfer theory methods, see e.g. Goel (1988). In this paper we introduce the radiosity method as a new tool to calculate the scattering from complex surfaces like plant canopies that may be described as three-dimensional structures.

The radiosity method describes an equilibrium radiation energy balance within a given enclosure that contains reflecting surfaces. It models the interaction of light between ideally diffuse (Lambertian) surfaces and computes the outgoing diffuse light intensity (radiosity) on all surfaces taking multiple reflections and transmissions into account.

The radiosity method was recently introduced as a new procedure for realistic image synthesis by Goral et al. (1984); it has its roots in thermal engineering, e.g. Hottel and Sarofim (1967), Siegel and Howell (1981). With the availability of fast computers and computer graphics algorithms, the radiosity method may become also a useful tool to solve optical remote sensing problems of structured 3-D surfaces. Most recently we discussed the basic principles of the radiosity method for canopy reflectance modeling (Gerstl and Borel (1990)) and compared its capabilities and underlying assumptions to those of the classical radiative transfer models in optical remote sensing. The purpose of this paper is to introduce the basic concept of radiosity to the remote sensing community, establish its relationship to radiative transfer, perform some sample calculations, and reference related work that has been done in computer graphics.

2 Radiosity

In this section we define the term radiosity, relate it to radiative transfer and show the connection to optical remote sensing.

2.1 The radiosity equation

The radiosity equation describes an equilibrium radiation energy balance within an enclosure that contains N discrete surfaces. It is assumed that all emission, transmission and reflection processes are diffuse (Lambertian). The light intensity leaving a surface element (radiosity) is composed of three components :

- Emitted light,
- Reflected light,
- Transmitted light.

A compact form of the radiosity relationship is given by Goral et al. (1984) and is also described by Greenberg (1989) for reflecting surfaces. The generalization for surfaces that are also transmitting radiation is straightforward and takes the following form for a differential surface element dS_i :

$$B_i \; dS_i \; = \; E_i \; dS_i + \chi_i \sum_{j=1}^{2N} \int_{S_j} B_j \; F_{dS_j \rightarrow dS_i} \; dS_j \quad , i = 1, 2, \ldots, 2N \tag{1}$$

where

$$\chi_i \; = \; \begin{cases} \rho_i, & \text{if } (\vec{n}_i \cdot \vec{n}_j) \; < \; 0 \\ \tau_i, & \text{if } (\vec{n}_i \cdot \vec{n}_j) \; > \; 0 \end{cases}$$

and

B_i **:** Radiosity of the differential area dS_i: the sum of emitted, reflected and transmitted radiative energy per unit time and area leaving a surface i, unit: $[Wm^{-2}]$. [1]

E_i **:** Emission of the differential area dS_i: the radiative energy per unit time and area emitted from a surface source, e.g. a light source within or on the surface, unit: $[Wm^{-2}]$.

ρ_i **:** Hemispherical reflectance of the differential area dS_i: the fraction of the hemispherically incident radiative flux which is reflected back into the top hemisphere surrounding surface i, unitless.

τ_i **:** Hemispherical transmittance of the differential area dS_i: the fraction of the hemispherically incident radiative flux onto the bottom surface which is transmitted through the surface into the hemisphere surrounding the top of surface i, unitless.

$\vec{n}_i$ **:** The normal vector on a surface i pointing outward.

$F_{dS_j \rightarrow dS_i}$ **:** View factor or form factor: the fraction of radiative energy leaving the infinitesimal surface dS_j that reaches the infinitesimal surface element dS_i, unitless.

N**:** Number of discrete surfaces (e.g. plant leaves), where 2N is the number of (single sided) surface components S_i.

Equation (1) states that the radiosity leaving a differential area dS_i is equal to the sum of emitted light plus reflected and/or transmitted fractions of radiosities transferred onto dS_i from all other surface elements dS_j and integrated over all theses other surfaces. Figure 1 shows the radiosity relationships , eq. (1), in a symbolic form.

[1] In the National Bureau of Standards nomenclature for reflectance by Nicodemus et al. (1977), the radiosity is called radiant exitance (M) and describes the exitant flux density.

2.2 The radiative transfer equation and its relation to radiosity

The classical approach to modeling the radiation field in a scattering and absorbing medium is based on the application of the radiative transfer (RT) equation which takes the form :

$$\mu \frac{\partial I}{\partial z} \; + \; \sigma_t \; I \; = \; \frac{\sigma_s^0}{2} \int_{(4\pi)} I(z, \underline{\Omega}') d\Omega' \; + \; Q(z, \underline{\Omega}) \tag{2}$$

for one-dimensional geometry and isotropic scattering (e.g. Liou (1980) or Gerstl and Zardecki (1985)), where

$I(z, \underline{\Omega})$ **:** Radiance, or specific intensity distribution function, at the spatial location z and into an infinitesimal cone around direction $\underline{\Omega}$, unit: $[Wm^{-2}sr^{-1}]$.

μ **:** Direction cosine $(\underline{\Omega} \cdot \underline{i}_z)$, unitless, where $\underline{i}_z$ is the normal vector into z direction.

σ_s^0, σ_t **:** Cross sections for isotropic scattering (σ_s^0) and the total of all interaction processes (σ_t) for each volume element ; $\sigma_t = \sigma_s + \sigma_a$ where σ_a is the absorption cross section, unit: $[m^2$ per m^3, or $m^{-1}]$.

$Q(z, \underline{\Omega})$ **:** Radiation emitted from an infinitesimal volume element at z into a cone around direction $\underline{\Omega}$, unit: $[Wm^{-3}sr^{-1}]$.

The RT equation (2) is a balance equation for radiative energy in an infinitesimal volume element at the location z; it equates the total energy flux that flows into such a volume element with that which flows out of it after undergoing changes due to scattering, absorption and emission (e.g. Liou (1980)).

Although both equations (1) and (2) express an equilibrium radiation energy balance, the radiosity equation (1) refers to a surface element i, while the RT equation (2) refers to a volume element. The connection between these different concepts is easily made by expressing the radiation flux J^+ (irradiance, exitance) that leaves the surface S_i (with normal vector $\underline{n}_i$) of a volume element in terms of the radiance $I(z_i, \underline{\Omega})$, cf. Nicodemus (1977), Liou (1980) :

$$B_i \; = \; J^+(z_i) \; = \; \int_{(2\pi)} (\underline{n}_i \cdot \underline{\Omega}) \; I(z_i, \underline{\Omega}) d\Omega \tag{3}$$

For an ideally diffuse, Lambertian surface the radiance distribution $I(z_i, \underline{\Omega})$ is uniform in all directions $\underline{\Omega}$, and the integral (3) can be evaluated analytically to yield

$$B_i \; = \; \pi \; I(z_i). \tag{4}$$

Note that the integration in eq. (3) is to be carried out only over the half-space of all outgoing directions, and

$$\int_{(2\pi)} (\underline{n}_i \cdot \underline{\Omega}) d\Omega \; = \; 2\pi \int_0^1 \mu \; d\mu \; = \; \pi. \tag{5}$$

With this relation between radiosity and radiance, eqs. (3) or (4), a mathematically rigorous derivation of eq. (1) from eq. (2) is possible but goes beyond the scope of this paper.

The conceptual differences between the radiosity and RT descriptions of the radiation distribution within a given volume of scattering and absorbing elements allows some important conclusions to be drawn. Consider, for the sake of argument, that our volume of interest is a plant canopy containing a distribution of leaves. To apply the classical radiative transfer method (RT-eq. (2)) for the computation of the radiation distribution within and on the boundaries of the canopy, the scattering and absorption properties of the leaves must be described by cross-sections σ_s and σ_t at every spatial location z within the canopy. To achieve this, the measurable canopy geometrical and optical parameters such as leaf density, size, shape, orientation, reflectance and transmittance, must be averaged over a volume element around z. It is standard procedure

Table 1: Comparison between the radiative transfer and radiosity methods

Radiative Transfer	**Radiosity**
Volume scattering	Surface reflection and transmission
Continuous medium	Discrete and oriented surfaces
Averaged scattering phase function	Explicit scattering characteristics
No spatial correlations of leaves	spatial correlations retained
No holes or clumps in canopy	Holes and clumps describable
Multiple scattering	Multiple scattering
Integro-differential equation	System of coupled linear equations

in canopy radiative transfer formulations (e.g. Gerstl and Zardecki (1985)) to relate measurable parameters like the leaf area index (LAI), leaf angle distribution (LAD), leaf hemispherical reflectance (ρ), transmittance (τ), and absorptance (α) to σ_s and σ_t and to derive the related quantities τ (optical depth) and ω (single scattering albedo). This averaging process eliminates the discrete nature of the leaves from the mathematical formulation and creates a continuous medium description. In a graphical formulation one might say that the real plant canopy with its finite number of leaves has been replaced by a gas-like medium whose radiative properties are equivalent to those of the real canopy in a volume-average sense, see Figure 2. Therefore, in the RT method the information about the location, shape and orientation of individual leaves is lost. Certain radiative effects of natural canopies that are based on the discrete nature of its components are no longer described by the RT equation. Such effects are, for example, those that are due to leaf size and shape, and the mutual shading or clumping of leaves. An example is the canopy hot spot (e.g. Gerstl, Simmer and Powers (1986)) which is not describable by a standard RT formulation.

In contrast, the radiosity method uses a set of $2N$ coupled equations of the form (1) which describe the radiative interactions between all N leaves of a plant canopy and retains the discrete nature of each leaf as a reflecting and transmitting surface S_i . Therefore, the information about the location, shape and orientation of all leaves is retained in the radiosity equations. The measurable reflection and transmission characteristics of the leaves enter the formulation directly. However, the number of coupled linear equations to be solved may be quite large, equal to twice the number of leaves in the canopy volume that is being considered. The geometrical correlations between individual leaves are described by the view factors F_{ij}. Therefore, the radiative effects due to the discrete nature of the leaves (shading, clumping, etc.) are fully described by the radiosity formulation. The irradiance on each individual leaf can also be computed. Multiple scattering between leaves is described by the radiosity equations, as it is contained as well in the RT equations. A schematic comparison between the special characteristics of the radiosity and RT methods is summarized in Table 1.

Thus, the radiosity method appears well-suited to calculate the radiation distributions within and at the boundaries of complex 3-dimensional assemblies of scatterers, such as a vegetation canopy. Since the discrete nature of leaves and their geometric correlations are maintained in the radiosity formulation, we expect it to provide an ideal complement to the existing radiative transfer formulations in canopy modeling.

2.3 Connection to Optical Remote Sensing

When remote sensing of a surface is considered, the quantity measured by a detector, which is at a given distance from that surface, is the radiance distribution $I_D(\lambda; x, y; \theta_v, \phi_v)$ emanating from surface points (x, y) and arriving at the detector in the view direction $\underline{\Omega}_v = (\theta_v, \phi_v)$, with wavelength λ. An intervening atmosphere between the surface and the detector, e.g. in the case of land remote sensing from satellites, will modify the emitted radiance distribution before it is received by the detector. For the present analysis we will disregard such atmospheric perturbations as well as specific detector characteristics and postulate an idealized point detector with infinitesimal field-of-view directly above the surface. In this idealized situation the objective of remote sensing is the determination of the radiance distribution function at the upper boundary of the surface, which may be a three-dimensional structure itself, like a vegetative canopy. The surface is assumed to be illuminated by a plane parallel radiation source, like the sun, with an incident irradiance $I_i(\underline{\Omega}_i)\cos\theta_i d\Omega_i$ from direction $\underline{\Omega}_i$ within the solid angle $d\Omega_i$, where θ_i is the angle between the incident direction and the surface normal. The radiance distribution that emerges from a point (x, y) of the illuminated surface into a reflected direction $\underline{\Omega}_r$ is then defined as (Nicodemus et al., 1977) :

$$dI_r(x,y,\underline{\Omega}_r) \;=\; f_r(x,y,\underline{\Omega}_i,\underline{\Omega}_r)\; I_i(\underline{\Omega}_i)\cos\theta_i \; d\Omega_i, \tag{6}$$

where $f_r(x,y,\underline{\Omega}_i,\underline{\Omega}_r)$ is the bidirectional reflectance distribution function ($BRDF$, unit $[sr^{-1}]$) that uniquely characterizes the reflection characteristic of the remotely sensed surface at each point (x, y) of its upper boundary. By measuring the reflected radiance distribution over each point (x, y) for several illumination directions and known incident irradiance, it is possible to approximate the $BRDF$ via eq. (6) by close-range remote sensing, when atmospheric effects can be neglected.

If the reflecting surface is a three-dimensional structure, like a vegetation canopy, it is also possible in principle to determine the BRDF by modeling the radiation field inside the structured surface to obtain the emerging radiance at the top boundary as a modeling result. This connection between remote sensing and canopy modeling has been exploited in the past by establishing radiative transfer models of greatly varying complexity, as has been summarized recently by Goel (1988). It is immediately apparent, however, that one-dimensional RT formulations such as eq. (2) cannot give a $BRDF$ at the top boundary that varies with the surface coordinates (x, y). A three-dimensional RT problem must be solved to achieve that goal. In addition, even if such 3-D RT-solutions were carried out, the effects of the discrete nature of leaves and stems would still not be included in the computed $BRDF$ distribution, as explained in the previous section. In contrast, the radiosity method can yield a spatially-dependent

$$BRDF \equiv f_r(x,y,\underline{\Omega}_i,\underline{\Omega}_r)$$

that also retains shading and clumping effects, while the radiosity equations do not grow in complexity. As shown later, after the radiosities are computed, it is possible and straightforward to create an image of the 3-D canopy model by rendering techniques borrowed from computer graphics. Thus, an experimental remote sensing scenario may be simulated closely. The effects of mutual shading of leaves on the canopy $BRDF$ under varying illumination and observation directions can typically lead to variations of the reflected radiance by up to a factor of 2, as measured for example by Kriebel (1977) for forests and grasslands.

3 View factors

A major problem in the application of the radiosity method is the calculation of the view or form factors F_{ij}. Repeating from subsection 2.1, the view factor F_{ij} is the fraction of the energy (radiant flux) leaving surface i and reaching surface j (unitless). For a closed environment with $2N$ surfaces, or in an open case (canopy) the sky and ground are also counted as surfaces, the sum of all view factors must be equal to one :

$$\sum_{j=1}^{2N} F_{ij} \;=\; 1 \quad , i = 1, 2, \ldots, 2N. \tag{7}$$

Note that the summation includes the term F_{ii} which represents the fraction of radiant energy leaving surface i that is incident on itself. This term is nonzero only if the surface patch i is concave (Hottel and Sarofim (1967)), which we exclude from our present formulation.

Three formulations of the view factors are commonly used (Sparrow and Cess (1978)):

a. Radiation transfer between infinitesimal surfaces. Consider two differential areas dS_i and dS_j at a distance r from each other. The normals to the surface elements are denoted by $\vec{n}_i$ and $\vec{n}_j$. The angles θ_i and θ_j are formed by the respective normals and the connecting line between the elements, compare figure 3.

The radiation flux leaving dS_i (radiosity) within a cone $d\Omega_i$ in direction of the element dS_j is given by

$$I_i dS_i \ \cos\theta_i \ d\Omega_i. \tag{8}$$

For the geometry depicted in Figure 3.a, it follows that if the cone $d\Omega_i$ is chosen so that it subtends ("sees") the surface element dS_j, we can write :

$$d\Omega_i \ = \ \frac{dS_j \ \cos\theta_j}{r^2}. \tag{9}$$

Relation (9) is a consequence of the purely geometrical requirements of flux conservation, where $dS_j \cos\theta_j$ is the projected area of the surface dS_j on a sphere with radius r centered around dS_i. Introducing (9) into the expression (8) and substituting I_i using eq. (4) one obtains

$$\frac{B_i \ \cos\theta_i \ \cos\theta_j \ dS_i \ dS_j}{\pi \ r^2}. \tag{10}$$

The ratio of the last expression to the total radiosity leaving dS_i, which is $B_i \ dS_i$, represents the fraction of the radiative energy leaving dS_i that arrives at dS_j. This fraction is the view factor

$$F_{dS_i \to dS_j} \ = \ \frac{\cos\theta_i \ \cos\theta_j \ dS_j}{\pi \ r^2}. \tag{11}$$

Using the same derivation as above, the view factor for diffusely distributed radiant energy leaving element dS_j and arriving at dS_i is given by

$$F_{dS_j \to dS_i} \ = \ \frac{\cos\theta_j \ \cos\theta_i \ dS_i}{\pi \ r^2}. \tag{12}$$

Comparing equations (11) and (12), one sees that :

$$dS_i \ F_{dS_i \to dS_j} \ = \ dS_j \ F_{dS_j \to dS_i}. \tag{13}$$

The relationship (13) is the reciprocity rule for radiant energy exchange between diffuse surfaces which is valid under two conditions :

1. The angular distributions of the radiances leaving the participating surfaces are Lambertian, i.e. constant in all directions.
2. For finite surfaces (see next two paragraphs), the magnitude of the emitted radiant flux density (radiosity) does not vary across the respective surfaces.

b. Radiation transfer between an infinitesimal and a finite surface. The view factor between an infinitesimal surface element dS_i and a finite surface S_j (see Figure 3.b) is obtained by integrating eq. (11) over surface S_j :

$$F_{dS_i \to S_j} \ = \ \int_{S_j} \frac{\cos\theta_i \ \cos\theta_j \ dS_j}{\pi \ r^2}. \tag{14}$$

The reciprocity relation for this case is

$$S_i \ F_{S_i \to dS_j} \ = \ dS_j \ F_{dS_j \to S_i}. \tag{15}$$

c. Radiation transfer between finite surfaces. The view factor between a finite surface S_i and a finite surface S_j (see Figure 3.c) is obtained by integrating eq. (11) over both surfaces (a detailed derivation is given in Sparrow and Cess (1978)):

$$F_{S_i \rightarrow S_j} = \frac{1}{S_i} \int_{S_i} \int_{S_j} \frac{\cos\theta_i \ \cos\theta_j \ dS_i \ dS_j}{\pi \ r^2}. \tag{16}$$

The reciprocity relation for this case is

$$S_i \ F_{S_i \rightarrow S_j} = S_j \ F_{S_j \rightarrow S_i}. \tag{17}$$

In equation (16) the view factor does not account for possible occlusion of parts of one surface by some object or another surface between S_i and S_j. In Goral et al. (1984) an additional term HID is introduced into equation (16) to account for occluding or hiding objects. The equation (16) then becomes

$$F^{HID}_{S_i \rightarrow S_j} = \frac{1}{S_i} \int_{S_i} \int_{S_j} \frac{\cos\theta_i \ \cos\theta_j \ HID \ \ dS_i \ dS_j}{\pi \ r^2}, \tag{18}$$

where the factor HID can assume values 0 (occlusion) and 1 (no occlusion) as a function of the direction vector $\underline{\Omega}_{i \rightarrow j}$ that connects the line of sight between the two surface elements dS_i and dS_j. The expressions (16) and (18) are difficult to evaluate for complex geometries and approximations have been developed (see subsection 4.3). View factor catalogs for simple geometries used in thermal engineering can be found in Hottel and Sarofim (1967); Siegel and Howell (1981); Welty (1976) and Sparrow and Cess (1978). Some view factors for specific shapes and orientations have been determined analytically. Examples of analytically solved cases include :

- Parallel directly opposed rectangles and disks,
- Infinitely long parallel cylinders,
- A sphere to a disk,

and some others, as given in the above references.

4 Algorithms to solve radiosity problems

A basic radiosity implementation on a computer includes the following five steps :

1. Generate the geometry describing a scene, i.e. the 3-D surface of interest.
2. Calculate the emission for all surfaces in the scene.
3. Compute the view factors between all participating surfaces.
4. Solve the radiosity equations.
5. Render the scene using the resulting radiosities for a given viewpoint.

In the following subsections each of the steps is described using an artificial vegetation canopy model as an example for a scene. The artificial canopy described is part of an ongoing experiment at the Los Alamos National Laboratory (Borel and Gerstl (1990)). The scattering of light from the artificial canopy has been modelled using raytracing and radiosity methods. The radiosity method described here can also be applied to other scenes.

4.1 Generation of the scene geometry

The scene geometry is a description of the environment in terms of

- polygons (list of vertices and list of edges),
- solid objects, such as a sphere, ellipsoid, tetrahedron, cube, cylinder, prism, etc.,
- reflectance and transmittance of the objects.

A description in terms of polygons is probably the easiest way, because the numerical solution of the radiosity method is based on solving a system of linear equations (see subsection 4.4). Solid objects can always be approximated as polygonal objects using standard computer graphics methods, e.g. Angell (1981). Cohen and Greenberg (1985) describe a hierarchical description of a scene used in an architectural radiosity problem. The algorithm uses adaptive subdivision of surfaces to account for rapid changes of the radiosity across neighboring surface patches.

4.2 Methods to compute the emission

In architectural interiors (see e.g. Goral et al. (1984); Cohen and Greenberg (1985); Cohen et al. (1986); Immel et al. (1986) and Nishita and Nakamae (1985)) the emission terms E_i are given by the energy per unit time and per unit area emitted by the light sources (emitting surfaces) in a room. All other (non-emitting) surfaces have an emission of zero. The radiosity method is then applied to solve for the radiant energy emerging from each surface (radiosity).

In our application to an "open" vegetative canopy, we consider all surfaces, or surface patches, that receive direct solar or sky radiation, as emitting surfaces. This implies that we must determine, by ray tracing for example, which surfaces or surface patches are illuminated. These illuminated surface patches then become the emission terms in the radiosity equations. The emission E on surface i at local coordinates (x, y) is given by :

$$E_i(x,y) \;=\; \chi_i \; HID(x,y) \; E_0 \; |\vec{n}_i \cdot \vec{s}|, \quad i = 1, \ldots, 2N, \tag{19}$$

where

$$\chi_i \;=\; \begin{cases} \rho_i, & \text{if } (\vec{n}_i \cdot \vec{s}) \;<\; 0 \\ \tau_i, & \text{if } (\vec{n}_i \cdot \vec{s} \;>\; 0 \end{cases}$$

and

$HID(x, y)$ **:** is the occlusion factor which can assume values 0 if another surface is in the direct line of sight between location (x,y) on the surface i and the sun (occlusion) and 1 (no occlusion) otherwise.

E_0 **:** is the incident solar flux.

$\vec{n}_i$ **:** is the normal vector on surface i and

$\vec{s}$ **:** is the vector pointing in the solar illumination direction.

Note if $(\vec{n}_i \cdot \vec{s}) \;>\; 0$ we also consider the transmitted solar radiation from an illuminated top surface as an emission term at the bottom of that surface. In the cases where a surface is partly illuminated we use the surface averaged emission E_i in the radiosity equations to limit the total number of surfaces to $2N$:

$$E_i \;=\; \frac{1}{S_i} \int_{S_i} E_i(x,y) \; dx \; dy, \; i = 1, \ldots, 2N, \tag{20}$$

where S_i is the area of the i-th surface. This averaging of the emission terms will consequently also result in surface-averaged radiosities B_i. We will later derive equations to recapture the measurable radiosities on sunlit and shaded areas.

4.3 Methods to compute the view factors

In Section 3 the definitions and equations for the three different types of view factors are established. Many methods exist to actually compute the view factors between finite surface patches (cf. Hottel and Sarofim (1967); Siegel and Howell (1981)), but we will use the unit sphere method.

To develop a deeper understanding of what a view factor is, we describe here the geometric analog for the view factor integral developed by Nusselt (1928), which is also called the unit sphere method to compute view factors. Consider a finite surface S_j as shown in Figure 4. Project the boundaries of the surface onto a hemisphere centered around the point where the view factor is to be evaluated. Then project the boundaries of the projected surface orthographically down onto the base of the hemisphere. The view factor is the fraction of this projected area to the total area of the circle which forms the base of the hemisphere.

Using a photographic method, one would take a picture of a scene containing all surfaces of interest, using an orthographic fish-eye lens. The view factor to a surface is determined by the fraction of its projection in the picture to the entire picture area. The projection onto the hemisphere takes the r^{-2} dependence into account. The orthographic projection has the effect of multiplying the area on the surface of the hemisphere by the cosine of the angle between the vector pointing to the projected surface and the normal on the receiving surface. The π in the denominator of equation (14) normalizes the area of the circle to 1. In Figure 5 a sample picture is shown using the orthographic fish-eye projection in an artificial canopy model (Borel et. al. (1989)). A view factor to any disk can be estimated by counting the number of pixels visible of the disk in the picture and dividing that number by the total number of pixels containing the projection of the hemisphere.

The hemi-cube method developed by Cohen and Greenberg (1985) is an efficient algorithm to compute view factors in complex environments. It takes advantage of well known algorithms in computer graphics (perspective projection, polygon clipping and depth-buffering) and is well suited for scenes made of polygons. Furthermore specialized hardware (frame buffers with high speed graphics accelerators) can be used to generate the view factors very rapidly. One computer workstation has become available only recently with radiosity libraries (e.g. Hewlett-Packard (1990)) for the progressive refinements method (Cohen et. al. (1988)).

From published execution times for radiosity programs, Cohen and Greenberg (1985) show that about 10 times more computer time is usually spent on the view factor calculation than on solving the radiosity equations or on rendering the scene.

Recently several researchers have pointed out some deficiencies of the Hemi-Cube method (e.g. Baum et al. (1989); Sillion and Puech (1989)) and proposed new methods. In our work we have developed a view factor algorithm which is based on Nusselt's geometrical analog that works very well for canopy radiosity calculations (see subsection 6.2).

4.4 Solving the radiosity equations

The radiosity equations (1) are a system of coupled simultaneous linear equations. Except for geometrically simple systems, eq.(1) is not analytically solvable. There are several methods which can be used to solve such integral equations numerically (Sparrow and Cess (1978)). Some of the methods have been applied to radiosity problems but it seems that for complex environments only one numerical method, the "Zone Method" developed first by Hottel and Sarofim (1967), has been applied successfully.

The radiosity equation (1) can be written for finite areas or patches if one can assume that the radiosity and emission terms remain constant over all finite surface patches S_i. This condition can in principle always be met by subdividing finite surfaces into sufficiently small patches :

$$B_i\ S_i \ = \ E_i\ S_i + \chi_i \sum_{j=1}^{2N} B_j\ F_{S_j \to S_i}\ S_j, \quad i = 1, 2, \ldots, 2N. \tag{21}$$

Using the reciprocity relationship for finite surfaces (17) the view factor from patch S_j to patch S_i can be written as :

$$F_{S_j \to S_i} \; = \; \frac{S_i}{S_j} \, F_{S_i \to S_j} . \tag{22}$$

Using eq. (22) in (21) and dividing both sides of the equation by S_i one obtains the basic radiosity relationship for finite area patches :

$$B_i \; = \; E_i + \chi_i \sum_{j=1}^{2N} B_j \; F_{ij} \quad , i = 1, 2, \ldots, 2N \tag{23}$$

where the simplified notation for the view factor $F_{ij} \; \equiv \; F_{S_i \to S_j}$ is used. The view factor F_{ij} is the fraction of radiant energy leaving patch i and reaching patch j. Equation (23) is a system of linear algebraic equations with known emissions E_i, $i = 1, 2, \ldots, 2N$, view factors F_{ij}, $i = 1, 2, \ldots, 2N$; $j = 1, 2, \ldots, 2N$ and reflection and transmission properties $\chi_i = (\rho_i, \; \tau_i)$. The unknown quantities are the radiosities B_i, $i = 1, 2, \ldots, 2N$. This system of equations can be written in matrix form :

$$\begin{bmatrix} (1 - \chi_1 \; F_{11}) & -\chi_1 \; F_{12} & \ldots & -\chi_1 \; F_{1,2N} \\ -\chi_2 \; F_{21} & (1 - \chi_2 \; F_{22}) & \ldots & -\chi_2 \; F_{2,2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\chi_{2N} \; F_{2N,1} & -\chi_{2N} \; F_{2N,2} & \ldots & (1 - \chi_{2N} \; F_{2N,2N}) \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{2N} \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{2N} \end{bmatrix}$$

or

$$[A] \; B \; = \; E \; . \tag{24}$$

Equation (24) could be solved by inverting matrix $[A]$ but this is not practical in our case, since the dimension of the matrix is usually quite large and an inversion would take at least $(2N)^3$ operations. Fortunately, iterative methods can also be used to solve eq.(24), since the matrix has a dominant diagonal and the off-diagonal terms are small or zero if occlusion occurs. The Gauss-Seidel method has been used most often in the solution of radiosity problems (Goral et al. (1984)).

4.5 Rendering of scenes

As a last step the computed radiosities are displayed in a process called rendering. An image of the scene can be rendered using standard computer graphics methods (Foley and Van Dam (1984)).

The geometrical parameters for the rendering are the

- Eye position, denoted by a vector $\vec{eye}$,
- View direction, $(\theta_v, \; \phi_v)$,
- Field of view, or the diagonal angle of view θ_d (e.g. 45^0 for an image taken with a standard photographic lens).

The image can be rendered using three different methods depending on the available hardware (Foley and Van Dam (1984)):

1. Z-buffer algorithm :
   (a) For each polygon calculate the polygon depth $z(x, y)$ and store it as a "z-buffer value"
   (b) If $z(x, y)$ is less than the current z-buffer value at (x, y), then :
      i. Place $z(x, y)$ into the z-buffer and
      ii. Place the radiosity value of the polygon at $z(x, y)$ into the "refresh buffer" at (x, y).

2. Depth sort algorithm :
   (a) Sort all polygons according to their largest z-coordinate of each polygon, then
   (b) Scan-convert each polygon in descending order of largest z-coordinate.
3. Ray tracing algorithm :
   (a) For each pixel (x, y) find the intersection with the closest object, then
   (b) Place the radiosity value of the polygon into the image buffer.

The first two algorithms are very fast if special graphics hardware is available to quickly "paint" polygons. The ray tracing algorithm is slow but works well if no special hardware is available. Workstations from Hewlett-Packard (1990) are now available which can render radiosity results in almost real time to allow an operator to change view position and view direction very quickly and "walk" through a scene or "fly over" it. Usually it is necessary to interpolate between adjacent radiosities to avoid a jump in intensity values across the picture. A bilinear interpolation is commonly used.

A very interesting and useful fact is that the radiosity solution is view independent, i.e. many different views can be rendered from different view points and view directions without repeating the costly radiosity or view factor solutions. The illumination conditions can be changed just by solving the radiosity equation (24) for another emission vector E. The radiosity equations can also be solved for various wavelengths by changing the reflectances and transmittances and the incoming light intensities to obtain spectrally variable (color) scenes.

5 Sample applications to remote sensing and comparisons with radiative transfer

The radiosity method has so far been used mostly to generate "pretty pictures" and to increase their realism (Cohen et al (1988)). The radiosity concept is, however, a physically correct model of how light interacts with diffuse surfaces and can therefore be viewed as a powerful tool to solve complex radiative transfer problems. Unlike the classical radiative transfer theory the radiosity method uses a geometrical description of the scene, through the view factors F_{ij}. For example, if a vegetative canopy can be described geometrically in every detail, then the radiosity method would produce an image that is comparable in every detail to a photographic picture or other remote measurements. Extensions of the radiosity method (see e.g. Immel et al. (1986); Wallace et al. (1987); Rushmeier and Torrance (1987) and Nishita and Nakamae (1987)), which include specular reflections and interactions of the light with the atmosphere, are expected to be very useful to calculate the amount of light which is detectable by satellite or airborne sensors.

5.1 Radiosity method applied to a layered vegetative canopy

Consider a plant canopy consisting of N discrete layers. Each layer consists of randomly arranged horizontal leaves with a leaf area index of lai per layer. The leaf area index of a layer is defined as the sum of its total area of leaves (one-sided) per unit surface area, unitless : m^2 of leaves per m^2 of surface. The leaves in a layer are assumed not to overlap each other and therefore $lai < 1$. The scattering from the leaves is assumed Lambertian, with a reflection coefficient ρ and transmission coefficient τ. A Lambertian ground surface is below the canopy and has a reflection coefficient ρ_s.

Note that this model is discrete in the vertical direction and quasi-continuous horizontally because the information about the location of the individual leaves in a layer is lost by defining the horizontally averaged quantity lai. We choose this hybrid model because an analogous radiative transfer model exists (see Ross and Nilson (1967)) where the vertical and horizontal leaf distributions are continuous. We will compare the solutions in subsection 5.2.

a. Radiosity Equations for a Single Layer above Ground. For a single layer canopy above a reflecting surface (see Figure 6) the three radiosity equations can be written down by heuristic arguments :

The radiosity B_1 on the top surface of the leaf layer is the sum of the emission E_1 and the transmitted part of radiosity B_3 from the ground surface, where the emission is treated as the reflection of direct solar radiation (compare also eq. (23):

$$B_1 = E_1 \; + \; \tau \; F_{13} \; B_3. \tag{25}$$

The radiosity B_2 on the bottom surface of the leaf layer is the sum of the emission E_2 and the reflected part of radiosity B_3 from the ground surface :

$$B_2 = E_2 \; + \; \rho \; F_{23} \; B_3. \tag{26}$$

The radiosity B_3 on the ground surface of the leaf layer is the sum of the emission E_3 and the reflected part of radiosity B_2 from the bottom of the leaf layer :

$$B_3 = E_3 \; + \; \rho_s \; F_{32} \; B_2 \tag{27}$$

The emission terms are those associated with the illumination source (sun) and depend on the reflectance, transmittance and the amount of shadowing due to the layer. For one homogenized layer canopy the emission from the top surface of the layer, averaged over the whole horizontal surface (leaves and gaps), is given by

$$E_1 \;\; = \;\; \rho \; lai \; E_0, \tag{28}$$

where E_0 is the total rate of energy flux incident per unit area in $[W \; m^{-2}]$. The incident energy flux on a horizontal leaf from a monodirectional radiation source,like the sun, is given by

$$E_0 \;\; = \;\; E_{sunlit} \cos \; \theta_i,$$

where E_{sunlit} is the incident flux from the sun (solar constant) and θ_i is the incidence angle for the light. Part of the incident flux is transmitted to the bottom surface of the leaf layer and, when averaged over the whole surface, the downward emission is given by

$$E_2 \;\; = \;\; \tau \; lai \; E_0. \tag{29}$$

In a horizontally averaged sense, the leaf layer shadows the ground surface by a fraction lai and is therefore transparent with a fraction $(1 - lai)$ of its surface. The upward emission from the ground surface, averaged over the whole ground, is then :

$$E_3 \;\; = \;\; \rho_s \; (1 - lai) \; E_0. \tag{30}$$

Using the principle of conservation of flux ($\rho + \tau + \alpha \;\; = \;\; 1$), the sum of the emitted and absorbed flux must be equal to E_0 for the entire single layer canopy :

$$\begin{aligned} E_0 &= E_1 + E_{layer} + E_2 + E_3 + E_{ground} \\ &= \rho lai E_0 + (1 - \rho - \tau) lai E_0 + \tau lai E_0 + \\ &\quad \rho_s (1 - lai) E_0 + (1 - \rho_s)(1 - lai) E_0 \\ &= lai E_0 + (1 - lai) E_0 \\ &= E_0 \; , \end{aligned} \tag{31}$$

where E_{layer} is the absorbed flux in the leaf layer, and E_{ground} is the absorbed flux in the ground.

The fraction of radiant flux leaving the ground surface and reaching the leaf layer (being intercepted by it) is given by the leaf area index lai, which is nothing else than the view factor between the ground surface and the leaf layer :

$$F_{13} \; = \; F_{23} \; = \; lai. \tag{32}$$

Another interpretation of eq. (32) can be found by computing the probability that a photon emitted from the ground surface or form the sun will be intercepted by a leaf. It can be shown that this probability is equal to lai for non-overlapping leaves (Ross, 1981).

Since the ground receives all of the energy or photons that leave the bottom of the leaf layer, we have

$$F_{32} \; = \; 1. \tag{33}$$

Using eqs. (28-30), (32)and (33) in eqs. (25-27) one can obtain:

$$\begin{aligned} B_1 &= \rho \; lai \; E_0 \; + \; \tau \; lai \; B_3 \\ B_2 &= \tau \; lai \; E_0 \; + \; \rho \; lai \; B_3 \\ B_3 &= \rho_s \; (1 - lai) \; E_0 \; + \; \rho_s \; lai \; B_2. \end{aligned} \tag{34}$$

The analytic solution of the set of linear equations (34) is :

$$\begin{aligned} B_1 &= E_0 \; lai \left[\rho + \tau \rho_s \frac{1 - lai(1-\tau)}{1 - \rho_s \rho lai} \right] \\ B_2 &= E_0 \; lai \left[\tau + \rho \rho_s \frac{1 - lai(1-\tau)}{1 - \rho_s \rho lai} \right] \\ B_3 &= E_0 \; \rho_s \frac{1 - lai(1-\tau)}{1 - \rho_s \rho lai}. \end{aligned} \tag{35}$$

To compare later this radiosity result with the radiative transfer result we must compute the upward and downward radiances from the radiosities. In Section 2.2 we derived the needed connection between radiosity and radiance (eq. (4)). Treating the leaf layer as a Lambertian surface the total upward radiance I^+ leaving the top side of the leaf layer is the sum of the radiances from the (horizontally averaged) leaves (B_1/π) and the contribution from the ground shining through the leaf layer $(1 - lai) \; B_3/\pi$ thus :

$$I^+ \; = \; \frac{1}{\pi}[B_1 \; + \; (1 - lai) \; B_3]. \tag{36}$$

The downward radiance I^- at the bottom of the leaf layer is given by :

$$I^- \; = \; \frac{1}{\pi} \; B_2. \tag{37}$$

$B_{1,2,3}$ in eqs. (36) and (37) may be substituted by the expressions (35) for a complete solution. Note that in a non-homogenized single leaf layer canopy with discrete leaves the radiance distribution I^+ is a function of direction and the x-y-location, and therefore eq. (36) would not hold.

b. Radiosity equations for N layers above ground. Figure 7 shows a cross section through this canopy model and labels the leaf layers and surfaces with the radiosities and their directions. Each layer is characterized by its leaf area index lai, the leaf reflectance ρ and the leaf transmittance τ. The ground has a reflection coefficient ρ_s.

Applying eq. (23), the radiosity at the top surface of the first layer, B_1, is the sum of the emission E_1 of layer 1 and the fractions of transmitted radiosities of the odd numbered surfaces below it :

$$B_1 \; = \; E_1 \; + \; \tau \; [F_{13} \; B_3 \; + \; F_{15} \; B_5 \; + \ldots + \; F_{1,2N-1} \; B_{2N-1} \; + \; F_{1,2N+1} \; B_{2N+1}]. \tag{38}$$

The solar emission term E_1 is given by eq. (28) as in the single layer canopy. The view factor F_{13} represents the fraction of intercepted photons coming from the top surface 3 of layer 2 to layer 1 which is equal to lai. The view factors for the deeper layers must take shadowing from layers above them into account. In a horizontally averaged sense the shadowing is due to interception of a fraction of lai photons per layer, therefore each layer lets a fraction of $(1 - lai)$ photons through. The view factor F_{15} is therefore equal to $lai\ (1 - lai)$. The view factor F_{17} is equal to $lai\ (1 - lai)^2$, etc. The view factor between the i-th and the j-th surface is therefore given by :

$$F_{ij} \ = \ lai\ (1 - lai)^{\frac{|i-j|}{2} - 1}, \quad i, j \ = \ 1, 3, 5, \ldots, 2N+1 \ , or\ i, j \ = \ 2, 4, 6, \ldots, 2N. \tag{39}$$

The radiosity B_2 on the bottom of the first layer is the sum of the emission term E_2, eq. (29), and the fractions of reflected radiosities from the odd numbered surfaces below it:

$$\begin{aligned} B_2 \ &= \ E_0\ \tau\ lai \ + \ \rho\ [lai\ B_3 \ + \ lai\ (1 - lai)\ B_5 \ + \ldots \\ &\ldots + \ lai\ (1 - lai)^{N-2}\ B_{2N-1} \ + \ lai\ (1 - lai)^{N-1}\ B_{2N+1}]. \end{aligned} \tag{40}$$

Similar formulas can be derived for all other leaf surfaces. The radiosity from the ground surface is the sum of the emission term and the fractions of the reflected radiosities of all even numbered surfaces above it :

$$\begin{aligned} B_{2N+1} \ &= \ E_0\ \rho_s\ (1 - lai)^N \ + \ \rho_s\ [(1 - lai)^{N-1}\ B_2 \ + \ (1 - lai)^{N-2}\ B_4 \ + \ \ldots \\ &\ldots + \ (1 - lai)\ B_{2N-2} \ + \ B_{2N}]. \end{aligned} \tag{41}$$

The radiosity equations (38-41) can be written in matrix form (24) where :

$$[A] = \begin{bmatrix} 1 & 0 & -\tau lai & 0 & -\tau lai(1 - lai) & \ldots & -\tau lai(1 - lai)^{N-1} \\ 0 & 1 & -\rho lai & 0 & -\rho lai(1 - lai) & \ldots & -\rho lai(1 - lai)^{N-1} \\ 0 & -\rho lai & 1 & 0 & -\tau lai & \ldots & -\tau lai(1 - lai)^{N-2} \\ 0 & -\tau lai & 0 & 1 & -\rho lai & \ldots & -\rho lai(1 - lai)^{N-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\rho_s(1 - lai)^{N-1} & 0 & -\rho_s(1 - lai)^{N-2} & 0 & \ldots & 1 \end{bmatrix},$$

$$[B] \ = \ \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ \vdots \\ B_{2N+1} \end{bmatrix} \quad ; \quad [E] \ = \ \begin{bmatrix} E_0\ \rho lai \\ E_0 \tau lai \\ E_0 \rho lai(1 - lai) \\ E_0 \tau lai(1 - lai) \\ E_0 \rho lai(1 - lai)^2 \\ \vdots \\ E_0 \rho_s (1 - lai)^N \end{bmatrix}. \tag{42}$$

It is important to remember that the computed radiosities are an average over the whole surface of a layer (including leaves and “holes” between them). The computed radiosities do not describe whether a certain leaf is in direct sunlight or is shaded by leaves above it.

Since the density of leaves in one layer and the probability of a photon reaching a leaf are known, it is possible to calculate the radiosities for a sunlit leaf B_i^{sunlit} and a shaded leaf B_i^{shadow} which would correspond to measurable quantities. Using the basic radiosity equations (23) one can see that the radiosity on the top surface of a sunlit leaf in layer i is the sum of the emission term $\rho\ E_0$ plus the contributions from reflected and transmitted radiosities from all other leaf layers and the ground :

$$B_{top,i}^{sunlit} = \rho\ E_0 \ + \ \frac{B_i \ - \ E_i}{lai}, \ i = 1, 3, 5, \ldots, 2N - 1, \tag{43}$$

where the B_i's are computed solving eq. (42) and the E_i's are given in eq. (42). The difference $(B_i - E_i)$ represents the sum of all reflected and transmitted radiosities from all other surfaces (the view factor summation term in eq. (23)) averaged over a leaf layer. To obtain the amount of energy flux on the discrete leaves one has to divide this term by the leaf area index lai because of the geometric averaging process that was used to formulate the radiosity equations (42). Each horizontally averaged layer intercepts the fraction lai of the total radiant flux at that layer position i in the canopy. However, in reality this light interception occurs only in the leaf surfaces, therefore the flux density on the individual leaves must be greater by a factor $\frac{1}{lai}$ than when averaged over the entire leaf layer. The radiosity $B^{sunlit}_{bottom,i}, i = 2,4,\ldots,2N$ for the bottom surface of a leaf, which is illuminated by the sun, is similar to eq. (43) except that the emission term is given by $\tau\ E_0$.

In the shadowed part on the top (or bottom) surface of a leaf only reflected or transmitted radiosities from other layers and the ground surface are contributing to the radiosity :

$$B_i^{shade} = \frac{B_i - E_i}{lai},\ i = 2,4,\ldots,2N. \tag{44}$$

Figure 8 shows the results for a radiosity calculation using eqs. (43) and (44) in a 10 layer canopy with a total leaf area index $LAI = 1.962$, leaf reflectance $\rho = 0.9$, leaf transmittance $\tau = 0.03$ and ground reflectance $\rho_s = 0.9$. The canopy parameters where chosen close to the measured parameters of our artificial canopy assembly. Note in Fig. 8, that there is an increase in the radiosity B^{sunlit}_{top} for the sunlit leaves in the layers below the first layer. We also observed this increase in our artificial canopy measurements (Borel and Gerstl(1990)). The increase is due to the additional light reflected downwards from layers above.

In order to compare the radiosity results with radiative transfer results it is necessary again to convert the radiosities into radiances which is possible since the computed radiances are average values over layers which can be treated as if they where Lambertian surfaces. The total radiance leaving a canopy layer is obtained by adding up the 'visible' radiosities from layers below or above the view position and dividing them by π, according to eq. (4). The visibility (view) factor of the nearest layer is 1 and for the k-th layer above or below, it is $(1 - lai)^{k-1}$. Figure 9 visualizes the contributions from various layers to the up/down radiances. The upward radiances at each layer are the weighted sum of the radiosities of layers below and divided by π. The upward radiances can be expressed in matrix form :

$$\begin{bmatrix} I_1^+ \\ I_2^+ \\ I_3^+ \\ \vdots \\ I_N^+ \end{bmatrix} = \frac{1}{\pi} \begin{bmatrix} 1 & (1-lai) & (1-lai)^2 & \ldots & (1-lai)^{N-1} \\ 0 & 1 & (1-lai) & \ldots & (1-lai)^{N-2} \\ 0 & 0 & 1 & \ldots & (1-lai)^{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix} \begin{bmatrix} B_1 \\ B_3 \\ B_5 \\ \vdots \\ B_{2N-1} \end{bmatrix}. \tag{45}$$

The downward radiances at each layer are the weighted sum of the radiosities of layers above and divided by π. The downward radiances can be expressed in matrix form :

$$\begin{bmatrix} I_1^- \\ I_2^- \\ I_3^- \\ \vdots \\ I_N^- \end{bmatrix} = \frac{1}{\pi} \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ (1-lai) & 1 & 0 & \ldots & 0 \\ (1-lai)^2 & (1-lai) & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (1-lai)^{N-1} & (1-lai)^{N-2} & (1-lai)^{N-3} & \ldots & 1 \end{bmatrix} \begin{bmatrix} B_2 \\ B_4 \\ B_6 \\ \vdots \\ B_{2N} \end{bmatrix}. \tag{46}$$

Equations (45) and (46) can be solved numerically after the radiosity equations (40-42) have been solved using the Gauss-Seidel iterative method (see section 6.5). Figure 10 shows the up/downward radiances for a radiosity calculation in a 10 layer canopy with a total leaf area index $LAI = 1.962$, leaf reflectance $\rho = 0.9$, leaf transmittance $\tau = 0.03$ and ground reflectance $\rho_s = 0.9$. Again, the canopy parameters are close to our artificial canopy. We also show results using the radiative transfer model by Ross (1981) which is described in the next section. Note that the radiative transfer solution is very close to the N-layer radiosity model.

5.2 Comparison of the radiosity solution with the radiative transfer solution

An analytical solution of the radiative transfer equation for a horizontal leave canopy was obtained by Ross (1981) for the following assumptions :

1. A Lambertian ground surface is located below the canopy with a reflection coefficient ρ_s.
2. The leaves are Lambertian with reflection ρ and transmission τ equal for both leaf sides.

Ross has shown that the up- and down radiances I^+ and I^- are independent of the direction of incident light and view direction if the radiative transfer formulation applies, and that the following differential equations hold

$$\frac{dI^+}{d\ell} = -\rho\, I^-(\ell) + (1-\tau)\, I^+(\ell) - \frac{\rho}{\pi}\, E_0\, e^{-\ell}, \tag{47}$$

$$\frac{dI^-}{d\ell} = -(1-\tau)\, I^-(\ell) + \rho\, I^+(\ell) + \frac{\tau}{\pi}\, E_0\, e^{-\ell}, \tag{48}$$

with the following boundary conditions :

$$I^-(\ell=0) = 0, \tag{49}$$

$$I^+(\ell=LAI) = \frac{\rho_s}{\pi}[E_0\, e^{-LAI} + \pi\, I^-(LAI)], \tag{50}$$

where $\ell = \ell(z)$ is the downward cumulative leaf area index and $LAI = \ell(z=H)$ is the total leaf area index of the canopy stand with height H. The downward cumulative leaf area index is defined as the integral of the (one-sided) leaf area index lai accumulated from the top of the canopy down to a depth z.

$$\ell(z) = \int_0^z lai(z')\, dz'\,. \tag{51}$$

The analytic solutions of eqs.(47) and (48) using the boundary conditions (49) and (50) are given in Ross (1981) and can also be found in Appendix A.

It can be show that the result of the radiosity method converges precisely to the radiative transfer formulation (eq. (47) through (50)) using the following steps :

1. Compute the difference between the radiances in layers n and $n+1$,
2. Let the leaf area index per layer lai, go to zero and the number of layers N go to infinity :

From the radiosity formulation of eq. (45) the upward directed radiance at layer n is given by :

$$\begin{aligned} I_n^+ &= \frac{1}{\pi}\,[B_n^+ + (1-lai)B_{n+1}^+ + (1-lai)^2 B_{n+2}^+ + \ldots + (1-lai)^{N-n-1}\, B_N^+] \\ &= \frac{1}{\pi}\, B_n^+ + (1-lai)I_{n+1}^+, \end{aligned} \tag{52}$$

where the notation B_n^+ describes the upward directed radiosity of layer n and is given by :

$$B_n^+ = \rho\, lai\, (1-lai)^{n-1}\, E_0 + \pi\, \tau\, lai\, I_{n+1}^+ + \pi\, \rho\, lai\, I_{n-1}^-. \tag{53}$$

Using eq. (53) in (52) one gets :

$$I_n^+ \;=\; [\tau\, lai \;+\; (1-lai)]\; I_{n+1}^+ \;+\; \rho\, lai\; I_{n-1}^- \;+\; \frac{\rho}{\pi}\, lai\; (1-lai)^{n-1}\; E_0. \tag{54}$$

Rearranging eq. (54) one finds :

$$\frac{I_n^+ \;-\; I_{n+1}^+}{lai} \;=\; -(1-\tau)\; I_{n+1}^+ \;+\; \rho\; I_{n-1}^- \;+\; \frac{\rho}{\pi}\; (1-lai)^{n-1}\; E_0. \tag{55}$$

Taking the limits $(I_n^+ \;-\; I_{n+1}^+) \;\rightarrow\; -dI^+,\; lai \;\rightarrow\; d\ell$ and $N \;\rightarrow\; \infty$, eq. (55) becomes the differential equation (47) for the upward radiance $I^+(\ell)$. Therefore in the limit the results from the radiosity method are identical to the results from the radiative transfer method (because the equations are identical).

From eq. (46) the downward directed radiance at layer n is given by :

$$\begin{aligned} I_n^- \;&=\; \frac{1}{\pi}\; [B_n^- + (1-lai)B_{n-1}^- + (1-lai)^2 B_{n-2}^- + \ldots + (1-lai)^{N-n-1}\; B_1^-] \\ &=\; \frac{1}{\pi}\; B_n^- + (1-lai) I_{n-1}^-. \end{aligned} \tag{56}$$

where the notation B_n^- describes the downward directed radiosity of layer n and is given by :

$$B_n^- \;=\; \tau\; lai\; (1-lai)^{n-1}\; E_0 \;+\; \pi\; \tau\; lai\; I_{n-1}^- \;+\; \pi\; \rho\; lai\; I_{n+1}^+. \tag{57}$$

Using the same steps as above one can also show that in the limit the differential equation for the downward radiance I^- from the RT formulation, eq. (42), can be derived from the radiosity formulation, eq. (46). We have also been able to derive a closed analytical solution for eqs. (45) and (46) which has the form

$$I_n^+ \;=\; C_1\; (1-b)^{n-1} \;+\; C_2\; (1+b)^{n-1}, \tag{58}$$

$$I_n^- \;=\; C_3\; (1-b)^{n-1} \;+\; C_4\; (1+b)^{n-1} \;-\; (1-lai)^n\; E_0, \tag{59}$$

where $C_1,\; C_2,\; C_3,\; C_4$ and b are complicated functions of $\rho,\; \tau,\; \rho_s,\; lai$ and E_0 (see Appendix B). Using much algebra one can show that eqs. (58) and (59) reduce in the limit to the radiative transfer results, eqs. (76) and (77) of Appendix A.

Figure 11 compares the numerical results of the radiosity and radiative transfer solutions for a 100 layer canopy with a total leaf area index of $LAI \;=\; 5.0$, leaf reflectance $\rho \;=\; 0.4$, leaf transmittance $\tau \;=\; 0.3$, ground reflectance $\rho_s \;=\; 0.5$ and incident energy flux $E_0 \;=\; 1$. The two results approximate each other very closely because the conditions $N \;=\; 100$ and $lai \;=\; 0.05$ are very close to the limit case of a homogenized canopy. The reflectances and transmittances were chosen to model a leaf in the near infrared regime. Both calculations reproduce the behaviour of observed radiation distributions in natural plant stands very truthfully (e.g. Ross (1980), Ross and Nilson (1967). Especially the fact that the averaged downwelling radiation flux first increases due to the high NIR transmission of leaves before it decreases at greater depths inside the canopy.

The N-layer radiosity method was implemented on a personal computer (IBM PC-AT with math-co-processor 8087). It took 70 sec to setup the matrix $[A]$ and the emission vector E for a 100 layer problem. In 132 sec the Gauss-Seidel method went through 22 iterations (one iteration takes about 6 sec) and converged to the solution with a convergence criterion of 10^{-9} for all radiosities.

6 Discrete leaf canopy modelling

In this section we describe the method used to generate a simulated vegetation canopy model on a computer, as well as a real artificial canopy structure, a new method to compute view factors, the algorithm to find the emission vector, a storage method to store very large view factor matrices and an iterative method to solve the radiosity equations. The N-layer model was used to test a radiosity program which computes the radiosities on discrete leaves. Once the radiosities are known, an image can be generated using a rendering method. By rendering images of canopy models for various sun and view directions using different leaf reflectances and transmittances, the spectral bidirectional reflectance distribution function (BRDF) can be obtained, as we shall show.

6.1 Generation of an artificial canopy geometry

The authors have designed and built an artificial simulated vegetation canopy consisting of over 10,000 circular disks of 5 cm diameter mounted in a 3 m x 3 m x 1.5 m structure (Borel and Gerstl (1990)). It was, therefore, natural to apply the radiosity method to such an artificial canopy and compare computed results with measured data. We used as basic surface elements circular disks for a simple leaf model. Each disk is described by a position vector pointing to the center of the disk $\vec{c}_i$, a normal vector $\vec{n}_i$ and a disk radius r_i, $i = 1, 2, \ldots, N_d$, where N_d is the number of disks. The reflectance ρ and transmittance τ are assumed to be the same for all surfaces. A computer program computes $\vec{c}_i$, $\vec{n}_i$ and r_i for a canopy with N_z layers of N_x x N_y leaves and a specified leaf angle distribution (e.g. planophile, plagiophile, erectophile, extremophile or uniform). We call a canopy model described by these parameters an artificial leaf canopy.

6.2 Computing the view factors for an artificial canopy

For a scene composed of disks we developed an algorithm based on raytracing because it seemed more efficient than the hemi-cube method introduced by Cohen et al. (1986). A summary of the algorithm used is given below :

1. Prepare the scene data as described in 6.1.
2. Select each disk as a view point and generate a fish-eye image using a very efficient ray tracing algorithm based on Nusselt's geometric analog (see section 4.3 and fig. 4).
3. The view factor to a disk is approximately given by the product of the number of pixels which cover a disk and the pixel area (see below), normalized to the total scene area.
4. Store the view factors in a file.

The above algorithm computes an approximation to the view factors because the integral given in eq. (18) was reduced to a summation of pixel areas dP for a given disk j as seen from the disk i, where dP is given by :

$$dP = \frac{A}{N_x\ N_y}, \tag{60}$$

where N_x and N_y are the number of pixels in x and y direction of the fish-eye picture with unit area (e.g radius $r = \frac{1}{\sqrt{\pi}}$) which has an square area $A = (2\ r)^2 = \frac{4}{\pi}$. The total number of pixels inside the fish eye is then :

$$N_{fisheye} = \frac{1}{dP} = \frac{\pi\ N_x\ N_y}{4}. \tag{61}$$

The accuracy of the view factors increases with the number of pixels in the fish-eye picture. In Figure 5 an example of such a fish-eye picture (for $N_x = N_y = 512$) from a leaf in the bottom layer of a 10 layer artificial canopy is shown. In the mean it takes about 2 or 3 intersection calculations for each pixel because our raytracing algorithm starts with intersections at the nearest disk. An algorithm without using any preprocessing would have to check for intersections with all other disks at each pixel, which would make the program execution very long. The improvement with the above algorithm is roughly given by the number of disks divided by the average number of intersection calculations (e.g. at 2 intersection calculations per pixel in a 2000 leaf canopy the improvement is a factor of 1000 times. A picture size of 50 x 50 pixels seems to be sufficient for most applications (Cohen and Greenberg (1985)). We found however that there are situations where a 100 x 100 fish eye picture was necessary to compute the view factors to a large number of distant discs (see Fig. 15). As a rule of thumb one should choose the fish eye picture resolution such that most of the surfaces have a view factor F_{ij} greater than dP or :

$$N_x \; N_y \; \geq \; \frac{4}{\pi \; F_{ij}}. \tag{62}$$

A doubling of the fish eye resolution resulted in a four fold increase of CPU time and 1.43 times more non-zero view factors. The view factor computer program was written in FORTRAN and executed on a DEC VAX 8600 with a CPU speed of 8 MIPS (Millions of instructions per second), 64 MBytes of core memory and 2.05 GBytes of hard disc storage. A 1000 leaf view factor problem took about 35 min of CPU time for a 50 x 50 pixel fish-eye picture. A 9000 leaf view factor program took 11 h 33 min of CPU time to complete and produced a view factor file of a size of 12.25 MBytes, storing 3.05 million non-zero view factors. In each fish-eye picture on the average 0.608 %, or 547 disks, were tested for intersections. Because of some preprocessing in the ray tracing algorithm (e.g. depth sorting and bounding box) the number of necessary intersection calculations was less than one intersection per pixel (0.949). Without bounding box and depth sorting the same problem would have taken 11.8 CPU years to complete. The same (not vectorized or otherwise optimized) view factor program was also run on a CRAY X-MP/48, a 80 MIPS machine, where it took about one fifth of the CPU time of the VAX 8600, about 2.3 h of CRAY CPU-time. It seems possible to achieve even better CPU-times by using dedicated special hardware and/or a massively parallel computer, like the Connection machine with 64,000 processors, or simply by vectorizing the computer code for CRAY execution.

Due to the large amount of CPU time and storage required for the view factor calculations, researchers at Cornell University (Cohen et al.,1986 and 1988) have devised a progressive refinement approach which allows a much faster but approximative calculation of the radiosities for scenes with up to 50,000 patches (surface elements) in 5 VAX CPU-hours but with a 'light gathering' step which took 190 CPU-hours on a VAX 8700. Most recent results by Wallace et al. (1989) show pictures with over 9000 polygons computed in 59 min on a Hewlett-Packard 9000 Series 835 TurboSRX workstation using a radiosity approach based on view factors from vertices to patches. It is not yet clear if these progressive radiosity methods can adequately model the light transport in vegetative canopies.

6.3 Computation of the emission terms

In the case of the artificial canopy we are interested in the radiosities on the disks due to direct sunlight and multiple reflections. Each disk, if illuminated by direct sunlight, acts like a light source and thus contributes to an emission term in the radiosity equations. A disk located in the top layer is always fully illuminated. A disk in a lower layer may be partially illuminated due to shading by disks in upper layers. To simplify the radiosity solution, it is assumed that the emission is uniformly spread out over each whole disk. This assumption is an approximation but using the subdivision property (see Siegel and Howell (1981)) for the view factors, one can show that the results are almost identical to the true case. The subdivision property states that a view factor to a surface patch can be written as the sum of view factors to smaller patches. The remaining problem is to find how much of each disk is illuminated by direct sunlight, which depends on the direction of the solar irradiation $\vec{s}\,(\theta_s, \; \phi_s)$. Assuming that the sun is a point source located at infinity one can use a ray tracing algorithm to compute an image of the artificial canopy as seen from the sun. The algorithm computes nearest intersections of rays $\vec{r}_{ij}(t)$ from the ij-th pixel (see Rogers (1985)) with a disk:

$$\vec{r}_{ij}(t) \;=\; \vec{a}_{ij} \;+\; t\;\vec{s}(\theta_s, \phi_s), \tag{63}$$

where t is the distance between the imaging plane and the some point along a line,

$\vec{a}_{ij}$ $= C\;\vec{s} \;+\; (i - M_x/2)\;du\;\vec{u} \;+\; (j - M_y/2)\;dv\;\vec{v}$ **;** a point in the imaging plane, which is the origin of a ray that is used to find intersections with disks,

C **:** a large negative number such that the imaging plane lies beyond the objects (disks),

$M_x,\; M_y$ **:** the number of pixels in horizontal and vertical direction in the raytraced picture which will be used to compute the emission terms on each disk,

$du,\; dv$ **:** the horizontal and vertical physical size of a pixel (e.g. in cm) and

$\vec{u}, \vec{v}$**:** unit vectors orthogonal to each other and to $\vec{s}$. The vectors $\vec{u}$ and $\vec{v}$ are used to compute the origin $\vec{a}_{ij}$ of the ray $\vec{r}_{ij}(t)$ and span the imaging plane.

The geometry for the ray intersection is shown in Fig. 12. The intersection of a ray $\vec{r}(t) \;=\; \vec{a} \;+\; t\;\vec{s}$ with a disk of radius r located at $\vec{c}$ with normal $\vec{n}$, is given by (see e.g. Rogers (1985)):

$$\vec{r}_{intersection}(t^*) \;=\; \vec{a} \;+\; t^*\;\vec{s}, \tag{64}$$

where :

$$t^* = \frac{\vec{n}\cdot\vec{c} \;-\; \vec{n}\cdot\vec{a}}{\vec{n}\cdot\vec{s}}$$

and

$$|\vec{c} \;-\; \vec{r}_{intersection}| \;<\; r.$$

Using eq. (20) one can estimate an average or "smeared-out" emission term $E_{smeared}$ for a disk which is given by the triple product of the total incident flux E_0 times the reflection (ρ) (transmission (τ)) coefficient times the fraction of projected disk area which is visible from the sun ($N_v\;du\;dv$), divided by the disk area ($\pi\;r^2$) and multiplied with the cosine of the angle between the disk normal and the sun direction ($|\vec{n}\cdot\vec{s}|$)

$$E_{smeared} \;=\; E_0\;\chi\;\frac{N_v\;du\;dv}{\pi\;r^2}\;|\vec{n}\cdot\vec{s}|, \tag{65}$$

where

E_0 is the incident solar irradiance, e.g. in $W\;cm^{-2}$,

χ is equal to the reflection coefficient ρ if $(\vec{s}\cdot\vec{n}) \;<\; 0$ and otherwise equal to the transmission coefficient τ,

N_v is the number of illuminated (visible from the sun) pixels covering the disk, and

$du \cdot dv$ is the area of a pixel (e.g. in cm^2).

6.4 An efficient method for storing the view factor matrix

For a canopy with $N \;=\; 10^4$ leaves the view factor matrix has $(2\;N)^2 \;=\; 4\cdot 10^8$ elements. Using a real number for each element with 4 bytes (64 bits) one would need

$$4\;(2\;N)^2 \;=\; 1.6\cdot 10^9 \;=\; 1.6\;Gbytes$$

of storage which is not practical on most available computer systems. An efficient storage method must be developed. It was found that the view factor matrix for an artificial canopy is very sparse: only 0.1 % to 10 % of all terms are non-zero. The sparseness is due to two effects :

1. Mutual occlusion of disks.
2. The algorithm summarized in section 6.2 considers only disks whose bounding boxes are larger than a pixel.

The sparseness of the view factor matrix depends also on the resolution of the fish-eye picture. As the individual pixel area decreases, more non-zero view factors appear in the matrix. It was found that the fraction of non-zero view factors is about $\alpha = 10\%$ in a 1000 leaf canopy for a fish-eye picture resolution of 50 x 50 pixels; for a canopy of 4000 leaves we found $\alpha = 1\%$ and a canopy of 9000 leaves has $\alpha = 0.1$ %.

The computed view factors are multiples of the pixel area dP as given by eq. (60), because the algorithm described in section 6.2 basically just counts the number of pixels per viewed disk area. Therefore we need to store only an integer number to represent a view factor which cuts the storage requirements in half since each integer can be stored in 2 *bytes*. From the matrix radiosity equation (24) one can see that the reflection or transmission coefficients are always multiplied with the view factors. Ideally one would like to solve the radiosity equations for different reflection and transmission coefficients, e.g. at different wavelengths and for different plant species. To retain this flexibility in the radiosity program we decided to only store the view factors and to use a look-up table for each set of reflection and transmission coefficients when the radiosity equations are solved (see section 6.5). For each view factor we must know, however, whether it has to be multiplied with the reflection or the transmission coefficient (see equation (1)). A simple indicator is to assign a positive integer for terms that have to be multiplied by ρ and a negative integer for terms to be multiplied by τ.

Because of the sparseness of the view factor matrix a significant amount of storage space can be saved if we count the number of zeros between non-zero view factors and encode them in a unique way. The method chosen is to add 10000 (which is much larger than the maximum possible number of $N_{fisheye} = 1963$ (eq.(61)) for a fish-eye image of 50 x 50 pixels resolution) to the number of zero terms encountered in a row of the view factor matrix to distinguish them from the actual view factor terms which have a much smaller range in values. Another significant savings was achieved by recognizing that the view factors from the top and the bottom surface components of a disk to another surface are identical (see eq. (42)), except that they are multiplied by either $-\rho$ or $-\tau$. Therefore, only every other row has to be stored.

Summarizing the sparse matrix encoding algorithm used for the view factor matrix in a pseudo notation:

1. For each element in a row of the view factor matrix do:
   (a) If the view factor needs to be multiplied by τ then use a negative sign for the view factor.
   (b) If the view factor is zero, then
   (c) $N_{zero} = N_{zero} + 1$ (N_{zero} is a counter for the number of zeros)
   (d) Else:
      i. If $N_{zero} = 0$ then store the view factor.
      ii. If $N_{zero} = 1$ then store 10000 plus the current view factor.
      iii. If $N_{zero} > 1$ then store $10000 + N_{zero}$ plus the current view factor.
      iv. $N_{zero} = 0$
   (e) End if.
2. End do.

Using this algorithm we were able to store a view factor matrix for 9000 leaves in only 12.25 MBytes instead of $4(2 \cdot 9000)^2 = 1.3$ GBytes, a savings of 99. %.

6.5 A fast iterative solution for the radiosity equations

The Gauss-Seidel iterative algorithm (see e.g. Dahlquist and Bjork (1974)) begins by using an initial guess for the solution. Assuming that the effects of multiple scattering are relatively small, a good first guess for B is then the emission vector E :

$$B_i^{(0)} = E_i \; . \tag{66}$$

The $(k+1)$-st iteration computes each radiosity B_j as a function of the already computed $B_j^{(k+1)}$ for $j = 1, \ldots, i-1$ and the previous $B_j^{(k)}$ for $j = i+1, \ldots, 2N$ as

$$B_i^{(k+1)} = E_i - \sum_{j=1}^{i-1} \chi_i \; F_{ij} \; B_j^{(k+1)} - \sum_{j=i+1}^{2N} \chi_i \; F_{ij} \; B_j^{(k)}, \quad i = 1, 2, \ldots, 2N. \tag{67}$$

The iteration is stopped when all absolute errors fulfill the criterion

$$| \; B_i^{(k+1)} - B_i^{(k)} \; | < \varepsilon, \quad i = 1, 2, \ldots, 2N. \tag{68}$$

If the compression algorithm for the sparse matrix (as described in subsection 6.4) is used to encode the view factor matrix, we save not only much storage space but also reduce the number of required multiplications in eq. (67). If a view factor F_{ij} is zero, or N_{zero} zeros are encountered, we simply increase j by 1 or N_{zero} and skip unnecessary multiplications with zero elements. The actual values for the matrix elements $\chi_i \; F_{ij}$ used in equation (67) can also be pre-computed and stored in an array A_m of length $2M$ with elements given by :

$$a_m = -\rho \; m \; dP, \quad m = 1, 2, 3, \ldots, N_{fisheye},$$

$$a_m = -\tau \; m \; dP, \quad m = -1, -2, -3, \ldots, N_{fisheye}.$$

During the iteration the non-zero view factors which are integers are used as indices $N_{fisheye}$ (eq. (61) in array A_m. This very efficient method solves the radiosity equations quickly. For example, for an artificial canopy with 1000 leaves, each iteration takes about 7 seconds CPU-time on a VAX-8600 and about 5 to 30 iterations depending on the scattering properties of the leaves and the specified iteration criterion ε (usually $\varepsilon = 10^{-9}$ is sufficient).

6.6 Rendering of artificial canopy scenes

No special hardware was available to render images of the artificial canopy using the Z-buffer or depth sort algorithms mentioned in subsection 4.5. Raytracing was used to render the radiosity results. A detailed description of the raytracing algorithm is discussed by Borel (1988).

In subsection 6.3 we indicated that the emission of a partly illuminated disk is uniformly spread over the whole disk surface. The radiosity found for each leaf surface ($2N$ top and bottom surfaces) by solving the radiosity equations is in effect the sum of the previously computed emission plus all additional reflected and transmitted radiosities from all other disks. The difference between the computed radiosity and the emission (see e.g. eq. (23)) represents the additional amount of light leaving the surface due to multiple reflections and transmissions from all other disks. In a sunlit part of a disk the radiosity B_i^{sunlit} is equal to

$$B_i^{sunlit} = \rho_i \; E_0 \; |\vec{n}_i \cdot \vec{s}| + \chi_i \sum_{j=1}^{2N} F_{ij} \; B_j. \tag{69}$$

For the part of the bottom surface of a sunlit disk a similar equation (69) holds except that the reflection coefficient ρ_i is replaced by the transmission coefficient τ_i in the first term and the χ's change from reflectance to transmittance and vice versa.

In a shaded part the only contribution to the radiosity is from light incident from other disks. The radiosity in the shadow B_i^{shadow} is therefore given by

$$B_i^{shadow} = \chi_i \sum_{j=1}^{2N} F_{ij}\, B_j. \tag{70}$$

Using eqs. (69) and (70) the radiosities in the illuminated and shadowed regions of a 10 x 10 x 10 leaf canopy were computed from the radiosity equation (23). The rendered canopy showing sunlit as well as shadowed parts of disks resulted in bright and dark areas with little apparent brightness variation in each. To make the small variations more visible on a monitor screen we used a contrast stretching algorithm, called histogram equalization (see e.g. Pavlidis (1982). Also, radiosities on sunlit leaves located near the edges of the canopy appeared slightly darker. The darker leaves are due to a smaller multiple scattering component since fewer leaves are found nearby and the radiation leaks out of the canopy sides.

The canopy model with discrete horizontal leaves can be used to compare the discrete leaf radiosity model with the horizontally homogenized N-layer radiosity model. To eliminate the canopy boundary effects that are realistically treated by the radiosity solution, it was necessary to only use radiosities from leaves near the center of the canopy for comparison. Figures 13.a (sunlit surface) and 13.b (shaded surface) show a comparison between the two radiosity calculations. The curves for the discrete leaf radiosity calculation for a 15 x 15 x 10 leaf canopy used only radiosities from leaves more than 5 leaves away (inside) from the boundaries to compute the leaf radiosities for the comparison. The incidence angle was 10 degrees from nadir and the incident flux $E_0 = 1.0$. The leaf reflectance was $\rho = 0.9$, the transmittance $\tau = 0.05$ and the ground reflectance $\rho_s = 0.9$. Some differences in the numerical results are due to the fact that the random locations of the leaves introduce a 'noise' in the components of the radiosity equations because only one realization of a computer generated random leaf distribution was considered. Averaging over many realizations of a random canopy and changing illumination directions will reduce these differences. Furthermore the resolution of the image to compute the emission terms may have introduced errors in the emission terms for the leaves in lower layers. Since the raytracing method to compute the emission terms failed to correctly compute the boundary condition of the ground surface was computed theoretically using the first term in eq. (41) multiplied with the cosine of the incidence angle.

In Figure 14 the rendered result of a discrete leaf radiosity calculation is shown (the exact computed gray levels can however not be reproduced in the figure) for a horizontally randomized artificial canopy model with 4000 horizontal disks (a) and a planophile maple leaf canopy with 4000 leaves (b). A natural maple leave was digitized using a charged coupled device (CCD) camera and a frame grabber. A modified intersection algorithm was used to decide if a ray intersection lies within the leaf shape or not. Thus we are able to compute radiosity solutions and render them for any leaf shape imaginable. The viewpoint was selected such that the center of the canopy and the observer lie in line with the illumination direction. This particular anti-solar view direction is also called the hot spot direction (see Gerstl, Simmer and Powers (1986) and Gerstl (1988)).

6.7 Computing the canopy bidirectional reflectance distribution function (BRDF)

The radiosity results of the discrete leaf or N-layer radiosity model are independent of any view direction and are isotropic on each leaf by definition. Only by rendering the canopy and using the radiosities for each pixel within a given field-of-view is it possible to generate an anisotropic angular variation. The rendering process takes mutual shading of leaves and the directional illumination of the leaves into account. For a given illumination direction and a certain observation direction the probabilities of seeing an illuminated or shaded leaf in a given canopy depth (layer) vary. These probabilities can be estimated analytically for simple geometries as shown by Gerstl, Simmer and Powers (1986). Therefore also the canopy reflectance into a certain observation direction varies greatly with the view direction. This angular canopy reflectance may be computed as an average radiance over a specified field-of-view.

From a picture that was rendered using parallel projection (see Fig. 12) one can find the approximate values of the bidirectional reflectance distribution function (BRDF) for a particular view direction (θ_r, ϕ_r) by computing the average of all pixel values p_k within a given detector field-of-view (FOV), which we assume to cover K pixels. The computed pixel values $p_k, k = 1, 2, \ldots, K$ are radiosities B_i for the nearest visible sunlit or shaded disk i or ground surface. Equations (69) and (70) can be used to derive the radiosities in the sunlit and shaded areas.

The radiance leaving the canopy in direction (θ_r, ϕ_r) is the average of all pixel values p_k divided by π for Lambertian leaves (see eq. (4)). The BRDF, as defined in eq. (6), is then given by the ratio of the radiance leaving the canopy in direction (θ_r, ϕ_r) over the incident irradiance $E_0(\theta_i, \phi_i)$,

$$f(\theta_i, \phi_i; \theta_r, \phi_r) \;=\; \frac{\frac{1}{K}\,\sum_{k=1}^{K}\,\frac{p_k}{\pi}}{E_0(\theta_i, \phi_i)}\;. \tag{71}$$

For a layered canopy model the results from the N-layer radiosity solution may be better suited to demonstrate a BRDF calculation than a discrete-leaf model, since there are no "noise" effects from a particular realization and no lateral boundary effects. Furthermore the rendering calculations take much CPU time and it proved more efficient to compute and store only the probabilities of seeing a sunlit $(P_m^{sunlit}(\theta_r, \phi_r))$ or shaded $(P_m^{shade}(\theta_r, \phi_r))$ leaf in a given layer m from direction (θ_r, ϕ_r). By multiplying the computed radiosities with the probabilities one has an alternate method to compute the average radiance leaving the canopy in direction (θ_r, ϕ_r) and we can compute the BRDF from

$$f(\theta_i, \phi_i; \theta_r, \phi_r) \;=\; \frac{1}{E_0(\theta_i, \phi_i)} \sum_{m=1}^{M} \;\frac{1}{\pi}\;[\;P_m^{sunlit}(\theta_r, \phi_r)\;B_m^{sunlit} \;+\; P_m^{shade}(\theta_r, \phi_r)\;B_m^{shade}\;]\;, \tag{72}$$

where the radiosities are given by eqs. (69) and (70) and N is the number of layers. The probabilities $P_m^{sunlit}(\theta_r, \phi_r)$ and $P_m^{shade}(\theta_r, \phi_r)$ are estimated by dividing the number of intersections with sunlit or shaded leaves in a given layer m by the total number of pixels of a rendered picture.

Solving the radiosity equations at different wavelengths for reflection coefficients $\rho = \rho(\lambda)$ and transmission coefficients $\tau = \tau(\lambda)$ one can obtain the spectral BRDF $f(\theta_i, \phi_i; \theta_r, \phi_r; \lambda)$ which does not require the recalculation of any view factors because they contain only geometrical (architectural) information that does not change with λ.

Using eq. (72), we computed the spectral BRDF for two wavelengths and several view directions in the principal plane, the plane that includes nadir and the illumination vector. The model canopy consisted of 20 x 20 x 10 randomized horizontal disks of 5 cm diameter with an average spacing of 10 cm in horizontal and vertical directions. The resulting BRDF "slice" is shown in Figure 15, where the sun direction is $\theta_i = 30^0$. The reflectance peak at $\theta_r = 30^0$ is the canopy hot spot (e.g. Gerstl (1988)). The leaf reflectances and transmittances were chosen to be typical for plants in the visible (VIS) and near infrared (NIR): $\rho_{VIS} = 0.09$, $\tau_{VIS} = 0.06$, $\rho_s = 0.4$, $\lambda_{VIS} = 0.4\ \mu m$; $\rho_{NIR} = 0.4$, $\tau_{NIR} = 0.3$, $\rho_s = 0.2$, $\lambda_{NIR} = 0.8\ \mu m$, confer Ross (1980).

Sixty five pictures, each measuring 200 by 200 pixels of the 4000- leaf canopy model were rendered for observation angles θ_r centered around discrete angles between 1 and 65 degrees from nadir. To simulate the soil, a large disk with a diameter of 2.82 m (the diagonal of the canopy model) was placed 10 cm below the 10-th (bottom) layer. The leaves of 5 cm diameter were all horizontal and spaced every 10 cm in x, y and z direction with a random horizontal offset of ± 2 cm in each layer. The image boundaries for the rendered pictures were selected such that only 25 % of the canopy was visible, e.g. for a range of view angles from 1 to 40 degrees only leaves in the top layer and below were visible but no canopy sides. For larger view angles (40 to 65 degrees) layers were visible from the canopy side and the large disk used to simulate the ground disappeared and many rays ended up in a black background which therefore reduced the BRDF values.

The hot-spot peak at $\theta_r = 30^0$ is due to the absence of canopy-internal shadows when the observation direction coincides with the solar direction : $\theta_r = \theta_i$ and $\phi_r = \phi_i$. Using the radiosity method and directional rendering the canopy hot spot could be obtained whereas the standard RT formulation (see section 2.2) is not able to obtain this angular signature. Earlier analyses have shown that the hot spot angular signature may be used to estimate the leaf size and other architectural parameters of plant canopies from aircraft and satellite remote sensing data (e.g. Gerstl, Simmer and Powers (1986) and Borel, Powers and Gerstl (1989)).

An important measure for the hot spot is the contrast ratio C :

$$C = \frac{f(\theta_i,\phi_i;\theta_i,\phi_i)}{f(\theta_i,\phi_i;\theta_r,\phi_r)}, \tag{73}$$

where the direction (θ_r, ϕ_r) is sufficiently far ($> 15^0$) away from the hot spot direction (θ_i, ϕ_i). Airborne observations have shown that the hot spot contrast ratio of healthy vegetation is smaller in the near infrared than in the visible. Our computations using the radiosity results (fig. 15) show that the contrast ratio for the NIR band is $C_{NIR} \approx 1.43$ which is indeed smaller than for the visible where we obtain $C_{VIS} \approx 1.56$ from this radiosity model calculation. This difference in the contrast ratio is due to the lighter shadows resulting from greater transmission and multiple reflection contributions in the near infrared band. Typical observed contrast ratios always lie below 2.5 because natural plant canopies are illuminated not only by direct sun light but also by diffuse sky light, scattered within the atmosphere, which we did not consider for this simple demonstration case. The effects of sky light may be included by adding a term $\chi_i\ B_{sky}\ F_{sky\ \rightarrow\ i}$ in eqs. (69) and (70), see Borel and Gerstl (1990).

We now compare the radiosity computed BRDF to a one-dimensional BRDF model based on probability theory. As shown by Gerstl, Simmer and Powers (1986) the probability $P(\theta_i;\theta_r)$ of seeing an illuminated leaf in a stratified and horizontally homogenous leaf canopy is given by :

$$\begin{aligned} P(\theta_i;\theta_r) \ = \ & \int_0^r \rho(z)\ \exp\{-\int_0^z \frac{\rho(u)}{r} du\}\ dz\ + \\ & \int_r^H \rho(z)\ \exp\{-\int_0^z \rho(u)\ du\}\ \exp\{-\int_r^z \rho(z)\ du\}\ dz\ , \end{aligned} \tag{74}$$

where H is the canopy height, $\rho(z) = \frac{d(l(z))}{dz}$, where $l(z)$ is the downward cumulative leaf area index, see eq. (51). The decorrelation depth r is given by

$$r = \frac{D}{|tan\theta_i - \tan\theta_r|}\ ,$$

and D is the leaf length in the 1-D model or the diameter for circular leaves.

Assuming single scattering is dominant, the canopy BRDF is approximated by :

$$f(\theta_i;\theta_r) = \rho_L\ P(\theta_i;\theta_r)\ , \tag{75}$$

where ρ_L is the hemispherical leaf reflectance. The BRDF slice in the principal plane using eq. (72) and eq. (75) was computed for $\rho = 0.09$, $\tau = 0.06$, $D = 5\ cm$, $H = 1\ m$ and $LAI = 1.962$. Both curves are compared in Fig. 16. The agreement between the BRDF based on raytracing and radiosity and the probabilistic model is very good considering that only one canopy realization was used to estimate the probabilities P in eq. (72). Additional comparisons, especially of computed BRDF's and measured values, are being reported in a separate paper by Borel and Gerstl (1990). A sample experimental result is described below.

In Fig. 17 a comparison between the 1-D BRDF model, the radiosity calculated BRDF and an actual measurement on a 10,092 disk artificial canopy is shown. An experiment using the artificial canopy (Borel and Gerstl (1990)) was performed. The artificial canopy consists of 12 layers with 29 x 29 circular disks in each layer. The leaf area index per layer is $lai = 0.1963$ and the layer spacing was 12 cm with an average horizontal disk spacing of 10 cm. White painted panels were used as a ground surface. The sun angle was 56 degrees from zenith. To reduce the finite size canopy boundary effects on the BRDF we decided to tilt the canopy 13 degrees towards the sun thus reducing the solar incidence angle to 42 degrees. The canopy was imaged from a boom mounted CCD camera from a distance of about 3.4 m above the top layer with a diagonal field of view of 15 degrees. A radial Fourier analysis (Borel and Gerstl (1989)) was performed around the shadow of the CCD camera head in an image of 300 x 300 pixels from 0 to 7 degrees in steps of 0.5 degrees relative to the retro-solar direction. One data point at 15 degrees was obtained by averaging a small area in the corner of the image. The hot spot curve shown in figure 17 was produced by mirroring the data and shifting the curve to a center angle of 42 degrees. The dip in the center is due to the shadow of the CCD camera. Asymmetries which exist at high solar zenith angles were thereby neglected. A Kodak Wratten filter (89B) (Kodak (1990)) was used to limit the spectral sensitivity to a band between 700 and 900 nm. The disk material has a band averaged reflectance of $\rho = 0.9$ and transmittance of $\tau = 0.01$ in this spectral band (Irons (1990)). Using the individual disk reflectance and image-derived intensity levels from the top layer disks in the hot spot image an approximate linear conversion between radial intensity values and radiosities was performed.

Using the same canopy parameters a radiosity-based BRDF was computed for a solar incidence angle of 42 degrees for viewing angles between 1 and 65 degrees in steps of 1 degree. An image of 200 x 200 pixels imaging a fraction of the whole canopy around the center of the top layer was used to compute the probabilities in eq. (72). The total CPU time for the BRDF calculation was 56 min. on a VAX 8600. An average of 4 intersection calculations per pixel were necessary to find the closest disk as described in section 6.3. The BRDF was computed using the raytracing derived-probabilities and the radiosity solution of a 12 layer problem with $lai = 0.1963$, $\rho = 0.9$ and $\tau = 0.01$.

The comparison in Fig. 17 shows good agreement of the measured HS signature near the hot spot direction (42^0). The disagreement on the left side is due to the mirroring of the measured HS curve which assumes angular symmetry. The analytical 1-D model follows the radiosity solution, but since only single scattering is considered in that model, its BRDF values away from the hot spot is lower than when the radiosity method is used. Again, the drop of the radiosity-based BRDF for large viewing angles ($\theta_r = 40^0, \ldots, 65^0$) is due to seeing more and more black background through the canopy sides. This drop is also seen in the measured canopy BRDF.

7 Strengths and weaknesses of the radiosity method for remote sensing applications

In this section we summarize briefly the pros and cons of the radiosity method in the context of its application potential to remote sensing of vegetative surfaces, and in comparison with other established analysis methods.

The largest apparent weakness of the radiosity method is its computer intensiveness, i.e. the large computer times and memory requirements. Clearly, the standard one-dimensional and semi-empirical canopy reflectance models, as described, for example by Goel (1988), are much less computer intensive, but also less realistic. A more equal comparison would be the radiosity method versus other three-dimensional models, such as the Monte Carlo ray tracing model by Ross and Marshak (1988), the geometric-optical model by Li and Strahler (1986), and 3-D radiative transfer models as, e.g. by Shultis and Myneni (1988). Computer requirements for all these models appear to be comparable to those of the radiosity method if similar canopy detail is being considered; on the other hand the radiosity method affords the fewest restrictive assumptions, such as volume-averaged canopy parameters (cf. sec. 2.2), planar boundary conditions, or simplified geometric structures such as cones, ellipsoids or cubes. The connection of the radiosity method with computer graphics through the view factor calculations and the rendering of results using the emerging field of scientific visualization has produced new and faster algorithms and hardware (Fuchs et. al. (1989)). The computer graphics community has also worked very hard to produce very realistic images of plants, trees and natural surfaces (Aono and Kunii (1984), Bloomenthal (1985), Borel (1988) and Prusinkiewicz (1988)). All these models are three dimensional and can be readily incorporated in the radiosity method for use by the remote sensing community. Photogrammetric methods have been developed to measure natural plant canopies (Lewis and Muller (1990)) that can also be used to generate the complex geometries encountered in nature.

It could also be argued that the time and effort required to perform a BRDF calculation with the radiosity method for a large 3-D canopy model that closely simulates a real canopy is excessive compared to the effort required to measure the BRDF in the field. Compared to tedious field measurements the radiosity method allows many parameter variations to be considered which can lead to a deeper understanding of the remote sensing signatures (spectral, angular and spatial) and their sensitivities. Measurements on natural canopies are also rarely reproducible due to the many uncontrollable influences of plant growth, wind, and other environmental factors.

It is clear that semi-empirical models may be computationally less demanding and one has to investigate whether the additional effort required for the 3-D canopy characterization in the radiosity method is worthwhile. It is possible to analyze with the radiosity method quantitatively the effects of heterogenous plant canopies, e.g. clumping of leaves, leaves with side-dependent reflectances, branches, fruits, flowers, senescent leaves, etc. Non-circular leaf shapes and non-planar leaf surfaces can also be treated.

There are only very few conceptual limitations for the radiosity method as compared to radiative transfer methods as discussed in sec. 2.2 and by Gerstl and Borel (1990). The fact that the radiosity method is intrinsically three-dimensional and allows consideration of as complex a geometrical arrangement of surfaces as is describable by computer graphics methods, makes it a valuable tool that extends existing modeling capabilities into the realm of near realism. It is clear that such expanded capabilities require some additional efforts in computer-time and memory; a judgement has to be made from case to case if the added computational effort is worthwhile. In this paper we present the radiosity method for Lambertian surfaces. In the literature one can find extensions to include non- Lambertian surfaces (Siegel and Howell (1981)) and descriptions of computer graphics implementations (Immel et al. (1986)). An extension of the radiosity method exists to include volumetric scattering and absorption, thus making it possible to study volume-surface interaction effects (Hottel & Sarofim (1967) and Rushmeier & Torrance (1987)). The additional refinements by including non- Lambertian and volumetric effects allow an even more realistic simulation of the scattering processes encountered in remote sensing, such as scattering by the atmosphere above the canopy. Coupling of these methods with conduction of heat and turbulent heat exchange may introduce new and exciting model descriptions that can be used to study global change.

The most apparent strength of the radiosity method is the capability of describing quantitatively and physically correct the interactions of radiation with many surfaces that may be arranged in a complicated structure. This is an advantage over the radiative transfer formulation where the discrete geometrical and optical canopy parameters such as leaf location, size, shape, and orientation must be averaged over volume elements to create a continuous medium description. Therefore, the radiosity method is expected to provide a valuable complement to the RT formulation in canopy reflectance modeling; it allows the analysis of radiative effects due to the discrete nature of leaves and stems and their heterogenous distributions. Examples of such effects are the mutual shading and clustering of leaves or stems and branches which can produce variations in BRDF values of up to factors of two as shown by field measurements, e.g. Kriebel (1977). Evaluating the canopy hot spot effect as shown in the previous section, is only one example for this strength inherent to the radiosity method. Clearly, the hot spot effect can be incorporated into the results of radiative transfer calculations, as e.g. done by Suits (1972), but it cannot be derived from these RT models, as described already earlier. Any description of the hot spot effect and mutual shading of leaves must be added retroactively to any RT model on the basis of some empirical knowledge; it is not derivable from first principles within the RT method. Therefore, the radiosity method is clearly a valuable extension beyond existing canopy reflectance models that are based on radiative transfer.

8 Summary and conclusions

The radiosity method, a concept to compute the transfer of radiative energy between surfaces, has been extended to the modeling of visible and near infrared radiation in vegetative plant canopies. Reflection and transmission of radiation from leaves within model canopies of up to 9000 discrete leaves have been considered. In a five-step calculation, the radiosity method provides a near realistic computer simulation of the entire remote sensing process from leaf detail to a complete canopy image. Individual leaf shapes and stems can be considered, as well as plant architectures that may introduce spatial and angular correlations between phytoelements, such as branching structures, leaf clumping, row effects, etc. The geometric/architectural information of a given plant canopy model is described and evaluated separately from the detailed radiosity computation by a view factor matrix that can be precomputed before the radiosity equations are solved. The radiosity method presented in this paper is limited to Lambertian leaves.

Although the radiosity concept originated in thermal engineering, it has most recently been used in computer graphics, and many computational algorithms have thus already been developed. Capitalizing on this knowledge base, the 52-year old Nusselt analogue was applied to compute view factors. An efficient storage scheme was developed for large view factor matrices. Thus the authors demonstrate that the computational requirements to solve the radiosity equations (and the concomitant view factor calculations) for up to 9000 reflecting and transmitting surface patches (leaves) can easily be met by todays minicomputers, such as a VAX 8600.

The radiosity solutions were validated against radiative transfer solutions for layered canopy models where closed analytical solutions are also possible. In the limiting case where the leaf area index per layer tends to zero while the number of leaf layers tends to infinity, i.e. for a homogenized, gas-like canopy model, the governing radiosity equations take a form identical to the radiative transfer equation developed earlier by Ross. For discrete leaf canopies an experimental verification against measurements on an artificial simulated canopy structure has also been performed but is reported in detail elsewhere.

The main advantage of the radiosity method compared to the radiative transfer method for canopy modeling is the capability to describe discrete leaf surfaces. It is particularly suited to model the angular reflectance signatures of vegetative canopies, such as the retro-reflectance peak, also called the canopy hot spot. Using modern computer graphics methods of rendering a computed radiosity scene, the computation of a BRDF "slice" along the principal plane of a 4000-leaf canopy model reproduces the canopy hot spot effect in great detail. This effect cannot be obtained from the classical radiative transfer models. The authors conclude, therefore, that the radiosity method provides a valuable complement to the existing radiative transfer methods for canopy modeling in the context of remote sensing analyses.

Many more remote sensing problems may be solved by the radiosity method, where surface to surface scattering occurs, e.g. scattering between opposing valley slopes to treat topography effects in remote sensing, non-linear mixing between vegetation and background signatures or the effects of cumulus clouds on satellite imagery. Plant biologists could apply the method to calculate light transport inside plant canopies under various illumination conditions and at different wavelengths to evaluate photosynthesis rates on discrete leaves. Extensions of the radiosity method are expected to include non-Lambertian surfaces and specular reflections, as well as coupled surface-atmosphere interactions. However substantial computer resources are needed to solve problems with 1000's of surfaces on a patch by patch basis. With the advent of high resolution imaging spectrometers such as HIRIS (Goetz et. al.(1987) and multi-directional satellite imagers like MISR (Diner et al (1989)), computational tools like the radiosity method are expected to be useful to simulate and interpret combined spectral, spatial and angular reflectance signatures, as described by Gerstl (1990).

Appendix A. Ross's radiative transfer result for a horizontal leaf layer canopy

The complete analytic solutions of eqs. (47) and (48) using the boundary conditions (49) and (50) have the form :

$$I^{+}(\ell) = \frac{E_0}{\pi} \frac{b_1 \, e^{-a \, (2 \, LAI - \ell)} - b_2 \, e^{-a \, \ell}}{a_1 - a_2 \, e^{-2 \, a \, LAI}}, \tag{76}$$

$$I^{-}(\ell) = \frac{E_0}{\pi} \frac{a_1 \, [e^{-a \, \ell} - e^{-\ell}] - a_2 \, e^{-2 \, a \, LAI} \, [e^{a \, \ell} - e^{-\ell}]}{a_1 - a_2 \, e^{-2 \, a \, LAI}}, \tag{77}$$

where :

$$\begin{aligned}
a &= \sqrt{(1 - \tau)^2 - \rho^2}, \\
a_1 &= \rho_s - \frac{1}{A'}, \\
a_2 &= \rho_s - A', \\
b_1 &= 1 - \frac{\rho_s}{A'}, \\
b_2 &= 1 - \rho_s \, A', \\
A' &= \frac{1 - \tau - a}{\rho}.
\end{aligned}$$

Appendix B. Analytic solution to the N-layer canopy radiosity problem

The analytic solution to the N-layer radiosity problem discussed in sections 5.1 and 5.2 is given by :

$$I_n^+ \ = \ C_1 \ (1-b)^{n-1} \ + \ C_2 \ (1+b)^{n-1}$$

$$I_n^- \ = \ C_3 \ (1-b)^{n-1} \ + \ C_4 \ (1+b)^{n-1} \ - \ (1-lai)^n \ E_0$$

where :

$$b = \frac{a \ lai}{2 \ \gamma} \ \{ \ \sqrt{(a \ lai)^2 \ + \ 4 \ \gamma} \ - \ a \ lai \} \ ,$$

$$a = \sqrt{(1-\tau)^2 - \rho^2} \ ,$$

$$\gamma = \tau \ lai \ + \ (1 - lai) \ ,$$

$$C_1 = \frac{E_0 \ \{ \ \rho \ lai \ t_2 (1+b)^{2N-3} \ + \ [2 \ g_3 \ g_4 \ b \ + \ \rho \ lai \ (1+b) \ t_1] \left[(1+b)(1-b)\right]^{N-2} \}}{c+d} \ ,$$

$$C_2 = \frac{C_1 \ g_2 \ - \ \rho \ lai \ E_0}{g_1} \ ,$$

$$C_3 = \frac{2 \ \rho \ lai \ C_1 \ b \ t_2 \ (1+b)^{N-2} \ + \ E_0 \ g_4 \ t_2 \ (1+b)^{N-2}}{g_1 \ (t_2 \ (1+b)^{N-2} + \ t_1 (1-b)^{N-2})} \ ,$$

$$C_4 = \frac{C_3 (1-b)^{N-2} \ t_1}{(1+b)^{N-2} \ t_2} \ ,$$

$$c \ = \ 2 \ \left[d(1-b^2)(\gamma - 1) + \gamma(1 - \gamma(1-b^2)) - 2 \ g_3 \ \rho \ lai \ b^2\right] \left[(1+b)(1-b)\right]^{N-2} \ ,$$

$$d \ = \ g_1 \ t_1 \ (1-b)^{2N-3} \ + \ g_2 \ t_2 (1+b)^{2N-3} \ ,$$

$$g_1 \ = \ \gamma(1+b) - 1 \ , \quad g_2 \ = \ 1 \ - \ \gamma(1-b) \ ,$$

$$g_3 \ = \ \gamma^2 \ \rho_s \ + \ \rho \ lai \ (1 - \rho \ \rho_s \ lai), \quad g_4 = \gamma^2 (1+b) - \gamma - (\rho \ lai)^2 \ (1+b) \ ,$$

$$t_1 = \gamma - (1-b)(1 - \rho \ \rho_s \ lai), \quad t_2 = (1 - \rho \ \rho_s \ lai)(1+b) - \gamma \ .$$

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Figure 1: Radiosity relations shown for a surface patch, confer eq.(1).

Figure 2: Concept of (a) the radiosity method and (b) the radiative transfer method.

Figure 3: View factors between (a) infinitesimal, (b) infinitesimal and finite and (c) finite surface patches, that are separated by a distance r.

Figure 4: Geometric analog for the view factor integral as developed by Nusselt (1928)

Figure 5: Computer generated fish-eye view from inside an artificial canopy of 20 x 20 x 10 horizontal disks as viewed from the 1811-th leaf, a leaf near the center of the bottom layer, looking up. A graphic representation of view factors, see text in sect. 4.3 and figure 4.

Figure 6: Radiosity formulation for a single layer with a leaf area index lai, leaf reflectance ρ and leaf transmittance τ above a ground with reflectance ρ_s.

Figure 7: Radiosity formulation for an N-layer canopy with a leaf area index per layer lai, leaf reflectance ρ and leaf transmittance τ, and a ground with reflectance ρ_s.

Figure 8: Radiosities on illuminated and shadowed parts of leaves for a 10 layer canopy with horizontal leaves and with a total leaf area index $LAI = 1.962$, leaf reflectance $\rho = 0.9$, leaf transmittance $\tau = 0.03$, ground reflectance $\rho_s = 0.9$ and incident flux $E_0 = 1.0$. The layer number can be computed by dividing the cumulative LAI by the layer $lai = 0.1962$.

Figure 9: Connection between radiosities and up (a) and downward (b) fluxes in a N-layer canopy.

Figure 10: Up- and downward radiances computed using the N-layer radiosity and radiative transfer solutions for a 10 layer canopy with a total leaf area index $LAI = 1.962$, leaf reflectance $\rho = 0.9$, leaf transmittance $\tau = 0.03$, ground reflectance $\rho_s = 0.9$ and incident flux $E_0 = 1.0$.

Figure 11: Comparison of the up- and downward radiances obtained by radiosity and radiative transfer methods for a 100 layer canopy with a total leaf area index $LAI = 5.0$, leaf reflectance $\rho = 0.4$,leaf transmittance $\tau = 0.3$, ground reflectance $\rho_s = 0.5$ and incident flux $E_0 = 1.0$.

Figure 12: Geometry for ray intersection with a disk

Figure 13: Comparison of the N-layer homogenized and discrete leaf radiosity solutions. The discrete leaf solution was obtained for a 20 x 20 x 10 leaf canopy. The radiosities on the sunlit (a) and shaded (b) leaf sides are shown for $E_0 = 1$.

Figure 14: Rendered hot spot picture of an artificial canopy with horizontal disks (a) with randomized offsets in x-y-z directions in a 20 x 20 x 10 grid. To illustrate that the ray tracing is possible for any leaf shape and leaf angle distribution, a planophile canopy of maple leaves (b) within a 20 x 20 x 10 grid was rendered.

Figure 15: Radiosity-computed BRDF in the principal plane for an artificial canopy with 4000 disks as a function of view angle for an illumination direction $\theta_i = 30\ deg$ for (a) visible ($\rho = 0.09$, $\tau = 0.06$) and (b) near infrared ($\rho = 0.4$, $\tau = 0.3$).

Figure 16: Comparison of the (a) radiosity calculated BRDF slice and (b) a single scattering analytical model (Gerstl et al (1988)) for $\rho = 0.09$, $\tau = 0.06$, $D = 5\ cm$, $H = 1\ m$ and $LAI = 1.963$.

Figure 17: Comparison of the (a) measured BRDF, (b) a radiosity calculated BRDF and (c) the single scattering analytical model, for $\theta_i = 42\ deg$,$\rho = 0.9$, $\tau = 0.06$, $D = 5\ cm$, $H = 1.44\ m$ and $LAI = 2.3556$.